



# Maple<sup>TM</sup>

# 10

Harnessing the Power of Mathematics

User Manual

# **Maple User Manual**

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# Maple User Manual

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# Preface

## The Maple Software

The Maple™ software is a powerful system that you can use to solve complex mathematical problems. You can also create professional quality documents, presentations, and custom interactive computational tools in the Maple environment.

You can access the power of the Maple computational engine through a variety of interfaces.

Interface	Description
Standard Worksheet	Full-featured graphical user interface offering features that help you create electronic documents that show all your assumptions, the calculations, and any margin of error in your results; or hide the computations to allow your reader to focus on the problem setup and final results. The advanced formatting features help you create the customized document you need. Because the documents are <i>live</i> , you can edit the parameters and, with the click of a button, compute the new results.  The <i>User Manual</i> was created using the Standard Worksheet interface to Maple. An interactive version of this manual is available in the Standard Worksheet interface. From the <b>Help</b> menu, select <b>Manuals, Dictionary, and more&gt;Manuals&gt;User Manual</b> .
Classic Worksheet	Basic worksheet environment for older computers with limited memory.
Command-line version	Command-line interface, without graphical user interfaces features, for solving very large complex problems or batch processing with scripts.
Maplesoft™ Graphing Calculator (Microsoft® Windows® only)	Graphical calculator interface to the Maple computational engine. Using it, you can perform simple computations and create customizable, zoomable graphs.

Interface	Description
Maplet™ Applications	Graphical user interface containing windows, textbox regions, and other visual interfaces, which gives you point-and-click access to the power of Maple. You can perform calculations and plot functions without using the worksheet or command-line interfaces.

This manual describes how to use the Standard Worksheet interface. Some features are not available in the Classic Worksheet interface and Command-line version. The Standard Worksheet interface has two modes: *Document* mode and *Worksheet* mode.

**Document Mode** - Using the Document mode, you can perform quick calculations. You can enter a mathematical expression, and then evaluate, manipulate, solve, or plot with a few keystrokes or mouse clicks.

**Worksheet Mode** - The Worksheet mode is designed for:

- Interactive use through Maple commands, which may offer advanced functionality or customized control not available using context menus or other syntax-free methods
- Programmatic use of the powerful Maple language

Using either mode, you can create high quality interactive mathematical presentations or documents.

## In This Manual

This manual provides an overview of all Maple features including:

- Performing computations
- Creating plots and animations
- Creating interactive documents
- The Maple programming language
- Using and creating custom Maplet applications

- File input and output, and using Maple with third party products
- Data structures

For a complete list of manuals, study guides, toolboxes, and other resources, visit the Maplesoft Web site at <http://www.maplesoft.com>.

## Audience

The information in this manual is intended for Maple users who have read the *Maple Getting Started Guide*.

## Conventions

This manual uses the following typographical conventions.

- **bold** font - Maple command, package name, option name, dialog, menu, and text field
- *italics* - new or important concept
- **Note** - additional information relevant to the section
- **Important** - information that must be read and followed

## Customer Feedback

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# 1 Document Mode

Using the Maple software, you can create powerful interactive documents. You can visualize and animate problems in two and three dimensions. You can solve complex problems with simple point-and-click interfaces or easy-to-modify interactive documents. You can also devise custom solutions using the Maple programming language. While you work, you can document your process, providing text descriptions.

## 1.1 Introduction

Maple has two modes: *Document* mode and *Worksheet* mode.

Using Document mode, you can quickly perform calculations. You can enter a mathematical expression, and then evaluate, manipulate, solve, or plot it with a few keystrokes or mouse clicks. This chapter provides an overview of Document mode.

Document mode sample:

Find the value of the derivative of  $\ln(x^2 + 1)$  at  $x=4$ .

$$\ln(x^2 + 1) \rightarrow 2\frac{x}{x^2 + 1} \rightarrow \frac{8}{17}$$

Integrate  $\sin\left(\frac{1}{x}\right)$  over the interval  $[0, \pi]$ .

$$\int_0^\pi \sin\left(\frac{1}{x}\right) dx = \sin\left(\frac{1}{\pi}\right) \pi - \text{Ci}\left(\frac{1}{\pi}\right)$$

Worksheet mode is designed for interactive use through commands and programming using the Maple language. The Worksheet mode supports the features available in Document mode described in this chapter. After review-

ing the information in this chapter, see Chapter 2, *Worksheet Mode (page 35)*, for information on using Worksheet mode.

Worksheet mode sample:

Find the value of the derivative of  $\ln(x^2 + 1)$  at  $x=4$ .

>  $\ln(x^2 + 1)$

$$\ln(x^2 + 1) \tag{1.1}$$

>  $\text{diff}((1.1), x)$

$$2 \frac{x}{x^2 + 1} \tag{1.2}$$

>  $\text{eval}((1.2), x=4)$

$$\frac{8}{17}$$

Integrate  $\sin\left(\frac{1}{x}\right)$  over the interval  $[0, \pi]$ .

>  $\int_0^\pi \sin\left(\frac{1}{x}\right) dx$

$$\sin\left(\frac{1}{\pi}\right) \pi - \text{Ci}\left(\frac{1}{\pi}\right)$$

**Important:** In every Maple document, you can use Document mode and Worksheet mode.

Using either mode:

- You have access to the full mathematical engine.
- You can create high quality interactive documents: easy-to-use computational tools, presentations, or publications.

Interactive document features include:

- Embedded graphical interface components, like buttons, sliders, and check boxes
- Automatic execution of marked regions when a file is opened
- Tables
- Character and paragraph formatting styles
- Hyperlinks

These features are described in Chapter 6, *Creating Mathematical Documents* (page 231).

**Note:** This chapter was created using Document mode. All other chapters were created using Worksheet mode.

## 1.2 In This Chapter

Section	Topics
Simple Mathematical Expressions - Introduction to Math and Text modes, and how to easily enter simple expressions	<ul style="list-style-type: none"> <li>• Text and Math Modes</li> <li>• Rational Expressions</li> <li>• Powers</li> <li>• Products</li> <li>• Shortcuts for Entering Mathematical Expressions</li> <li>• Other Expressions</li> </ul>

Section	Topics
Evaluating Expressions - How to evaluate expressions	<ul style="list-style-type: none"><li>• Displaying the Value Inline</li><li>• Displaying the Value on the Following Line</li></ul>
Editing Expressions and Regenerating Output - How to update expressions and results	<ul style="list-style-type: none"><li>• Updating a Single Computation</li><li>• Updating a Group of Computations</li><li>• Updating All Computations in a Document</li></ul>
Entering Expressions - Overview of tools for creating complex mathematical expressions	<ul style="list-style-type: none"><li>• Palettes</li><li>• Symbol Names</li></ul>
Performing Computations - Overview of tools for computing and plotting	<ul style="list-style-type: none"><li>• Computing with Palettes</li><li>• Context Menus</li><li>• Assistants and Tutors</li></ul>
Document Mode Summary - Summary of key Document mode features	<ul style="list-style-type: none"><li>• Table of Document Mode Tools</li></ul>
Getting Help - A list of resources available in the Maple Help System	<ul style="list-style-type: none"><li>• Table of Maple Help Resources</li></ul>

## 1.3 Simple Mathematical Expressions

In Document mode, you can enter two types of content: *Text* and *Math*. The **Text** mode and **Math** mode icons at the left end of the toolbar indicate the current mode. The toolbar is located near the top of the Maple window, immediately below the menu bar.

To switch between Text and Math modes, press the **F5** key. (Alternatively, click the **Text** mode or **Math** mode toolbar icon.) Consequently, it is easy to enter sentences containing text and inline mathematical expressions.

Entering mathematical expressions, such as  $\frac{35}{99} + \frac{1}{9}$ ,  $x^2 + x$ , and  $xy$ , is natural in Math mode.

## Rational Expressions (Fractions)

### To enter a fraction:

1. Enter the numerator.
2. Press the forward slash (/) key.
3. Enter the denominator.
4. To exit the denominator, press the right arrow key.

## Powers

### To enter a power:

1. Enter the base.
2. Press the caret (^) key (**Shift** + **6**).
3. Enter the exponent, which displays in math as a superscript.
4. To exit the exponent, press the right arrow key.

## Products

### To enter a product:

1. Enter the first factor.
2. Press the asterisk (\*) key, which displays in math as  $\cdot$ .
3. Enter the second factor.

## Implied Multiplication

In most cases, you do not need to include the multiplication operator,  $\cdot$ . Insert a space character between two quantities to multiply them.


**Note:** In some cases, you do not need to enter the multiplication operator or a space character. For example, Maple interprets a number followed by a variable as multiplication.

## Shortcuts for Entering Mathematical Expressions

Table 1.1 lists shortcut keys for entering and navigating mathematical expressions.

**Table 1.1: Shortcuts for Entering Mathematical Expressions**

Symbol/Format	Key	Automatically Generated in Document
Switch between Math and Text modes	F5	Example using fraction: $\frac{1}{4}$ (Math) versus 1/4 (Text)
Fraction	/ (forward slash)	$\frac{1}{4}$
Exponent	^ (caret)	$x^2$
Subscript	_ (underscore)	$x_a$
Overscript	<ul style="list-style-type: none"> <li>• <b>Ctrl + Shift + "</b>, Windows and UNIX®</li> <li>• <b>Command + Shift + "</b>, Macintosh®</li> </ul>	$\overset{\rightarrow}{x}$
Square root	Enter <b>sqrt</b> , and then press the command/symbol completion shortcut. (See the following row in this table.)	$\sqrt{25}$

Symbol/Format	Key	Automatically Generated in Document
Command/symbol completion	<ul style="list-style-type: none"> <li>• <b>Ctrl + Space</b>, Windows</li> <li>• <b>Command + Shift + Space</b>, Macintosh</li> <li>• <b>Ctrl + Shift + Space</b>, UNIX</li> </ul>	$ab$ 
Navigating an expression	Arrow keys	

For a complete list of shortcut keys, refer to the **Math Shortcut and Hints** help page. To access this help page in the Maple software, in Math mode enter **?MathShortcuts** and then press **Enter**. For information on the Maple Help System, see *Getting Help (page 32)*.

## Other Expressions

It is also easy to enter mathematical expressions, such as:

- Piecewise-continuous functions:  $|x| = \begin{cases} -x & x < 0 \\ 0 & x = 0 \\ x & x > 0 \end{cases}$

- Limits:  $\delta(x) = \lim_{\epsilon \rightarrow 0} \epsilon |x|^{\epsilon - 1}$

- Continued fractions:  $\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$

and more complex expressions. For information, see *Entering Expressions (page 10)*.



## 1.4 Evaluating Expressions

To evaluate a mathematical expression, place the cursor in the expression and press **Ctrl** + = (**Command** + =, for Macintosh). That is, *press and hold* the **Ctrl** (or **Command**) key, and then press the equal sign (=) key.

To the right of the expression, Maple inserts an equal sign and then the value of the expression.

$$\frac{2}{9} + \frac{7}{11} = \frac{85}{99}$$

You can replace the inserted equal sign with text or mathematical content.

### To replace the equal sign:

1. Select the equal sign. Press **Delete**.
2. Enter the replacement text or mathematical content.

For example, you can replace the equal sign with the text "is equal to".

$$\frac{2}{9} + \frac{7}{11} \text{ is equal to } \frac{85}{99}$$

In mathematical content, pressing **Enter** evaluates the expression and displays it centered on the following line. The cursor moves to a new line below the output.

$$\frac{2}{9} + \frac{7}{11}$$

$$\frac{85}{99}$$

By default, Maple labels output that is generated by pressing **Enter**. For information on equation labels, see *Equation Labels* (page 59). In this manual, labels are generally not displayed.

In text, pressing **Enter** inserts a line break.

You can use the basic algebraic operators, such as  $+$  and  $-$ , with most expressions, including polynomials—see *Polynomial Algebra (page 126)*—and matrices and vectors—see *Linear Algebra (page 135)*.

$$(2x^2 - x + 1) - (x^2 + 2x + 12) = x^2 - 3x - 11$$

$$3 \cdot \begin{bmatrix} -4 & 8 & 99 \\ 27 & 69 & 29 \end{bmatrix} = \begin{bmatrix} -12 & 24 & 297 \\ 81 & 207 & 87 \end{bmatrix}$$

## 1.5 Editing Expressions and Updating Output


One important feature of Maple is that your documents are *live*. That is, you can edit expressions and quickly recalculate results.

### To update one computation:

1. Edit the expression.
2. Press **Ctrl** += (**Command** +=, for Macintosh) or **Enter**.


The result is updated.

### To update a group of computations:

1. Edit the expressions.
2. Select all edited expressions and the results to recalculate.
3. Click the Execute toolbar icon .

All selected results are updated.

### To update all output in a Maple document:

- Click the Execute All toolbar icon .

All results in the document are updated.

## 1.6 Entering Expressions

Mathematical expressions can contain the following symbols.

- Numbers: integers, rational numbers, complex numbers, floating-point values, finite field elements,  $i$ ,  $\infty$ , ...
- Operators:  $+$ ,  $-$ ,  $!$ ,  $/$ ,  $\cdot$ ,  $\int$ ,  $\lim_{x \rightarrow a}$ ,  $\frac{\partial}{\partial x}$ , ...
- Constants:  $\pi$ ,  $e$ , ...
- Mathematical functions:  $\sin(x)$ ,  $\cos(\pi/3)$ ,  $\Gamma(2)$ , ...
- Names (variables):  $x$ ,  $y$ ,  $z$ ,  $\alpha$ ,  $\beta$ , ...
- Data structures: sets, lists, Arrays, Vectors, Matrices, ...

Maple contains over a thousand symbols. For some numbers, operators, and names, you can press the corresponding key, for example, **9**, **=**, **>**, or **x**. Most symbols are not available on the keyboard, but you can insert them easily using two methods.

To insert a symbol, you can use:

- Palettes
- Symbol names

### Palettes

Palettes are collections of related items that you can insert by clicking or dragging. Palettes contain:

- Numbers and constants, like **i** (the imaginary unit),  $\pi$ , and  $\infty$ . For example, see the **Common Symbols** palette (Figure 1.1).

- Layouts, like an item with a superscript and subscript. For example, see the **Layout** palette (Figure 1.2).
- Mathematical operations, like a definite integral with placeholders for the integrand, variable of integration, and endpoints of the interval of integration. For example, see the **Expression** palette (Figure 1.3).
- Specialized tools. For example, see the **Matrix** palette (Figure 1.4). For information on the **Matrix** palette, see *Creating Matrices* (page 135).



Figure 1.1: Common Symbols Palette

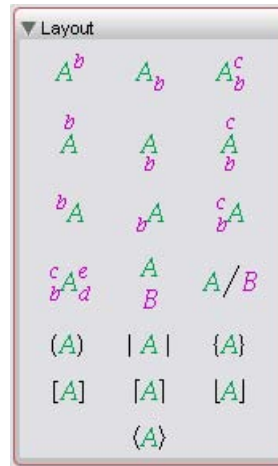


Figure 1.2: Layout Palette

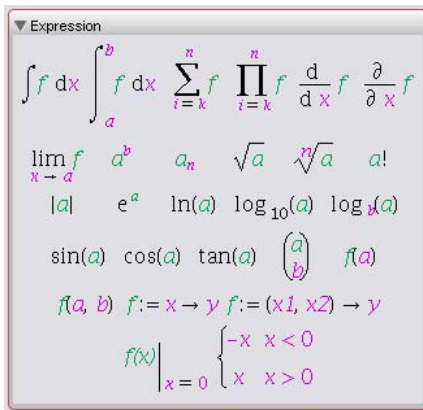


Figure 1.3: Expression Palette

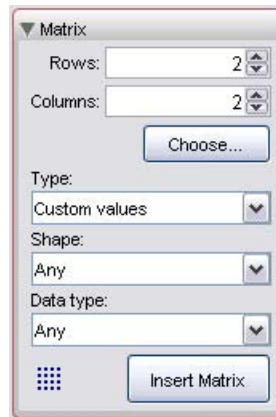


Figure 1.4: Matrix Palette

## Using Palettes

### To insert a palette item:

1. In the palette, click the item to insert. The item is inserted at the cursor location.
2. If the item has colored placeholders, specify values for them.
  - To move to the next placeholder, press the **Tab** key.

**Note:** You can drag palette items to any location in the document.

### For example, to insert the constant $\pi$ :


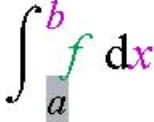
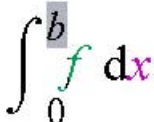
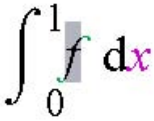
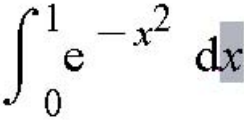
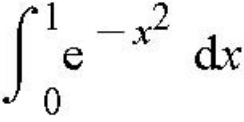
- In the **Common Symbols** palette, click the  $\pi$  symbol.

or

- From the **Common Symbols** palette, drag the  $\pi$  symbol to the appropriate location in the document.

Table 1.2 shows how to enter a definite integral.

**Table 1.2: Entering a Definite Integral**

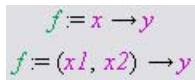
Action	Result in Document
1. In the <b>Expression</b> palette, click the definite integration item  . Maple inserts the definite integral. The left endpoint placeholder is selected.	
2. Enter <b>0</b> , and then press <b>Tab</b> . The right endpoint placeholder is selected.	
3. Enter <b>1</b> , and then press <b>Tab</b> . The integrand placeholder is selected.	
4. Enter $e^{-x^2}$ , and then press <b>Tab</b> . The variable of integration placeholder is selected.	
5. Enter <b>x</b> .	

To evaluate the integral, press **Ctrl+=** (**Command+=**, for Macintosh) or **Enter**. For more information, see *Computing with Palettes* (page 20).

## Defining a Mathematical Function

### To define a function of one or two variables:

1. In the **Expression** palette, click one of the function definition items (Figure 1.5). Maple inserts the function definition.
2. Replace the placeholder **f** with the function name. Press **Tab**.
3. Replace the parameter placeholders, **x** or **x1**, **x2**, with the independent variable names. Press **Tab**.
4. Replace the final placeholder, **y**, with the expression that defines the function value. Press **Enter**.



**Figure 1.5: Function Definition Palette Items**

For example, define a function that doubles its input.

$$twice := x \rightarrow 2x$$

$$x \mapsto 2x$$

$$twice(1342) = 2684$$

$$twice(y - z) = 2y - 2z$$

**Note:** To insert the right arrow symbol  $\rightarrow$ , you can also enter the characters `->`.

**Important:** The expression  $2x$  is different from the function  $x \rightarrow 2x$ .

For more information on functions, see *Functional Operators* (page 292).

## Viewing and Arranging Palettes

By default, palettes are displayed in palette docks at the sides of the Maple window. If no palette dock is visible, use the following procedure.

### To view palette docks:

- From the **View** menu, select **Palettes**, and then **Expand Docks**.

### To expand a palette in a palette dock:

- Click the triangle at the left of the palette title.

### To move a palette in a palette dock:

- Drag the palette (by clicking its title) to the new location.

## Adding Palettes to the Palette Docks

Maple has over 20 palettes. By default, only a few palettes are in the palette docks. To add a palette to a palette dock, use the following procedure.

### To add a palette:

1. Right-click (**Control**-click, for Macintosh) a palette dock. Maple displays a context menu—a menu that lists actions you can perform on the object—near the palette.
2. From the context menu, select **Show Palette**, and then select the palette.

## Symbol Recognition Palette

Finding the right symbol to insert can be time consuming. The **Symbol Recognition** palette provides an efficient way to find and insert the right symbol. You draw the symbol with your mouse and then Maple matches your input against items available in the system. See Figure 1.6.



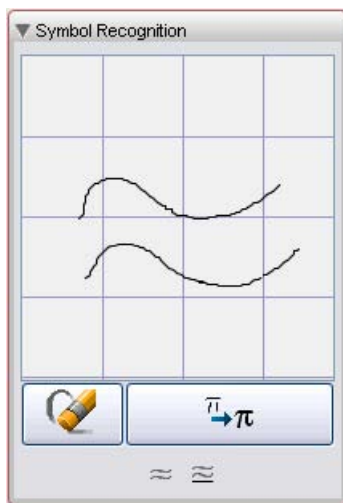
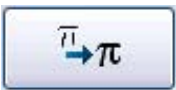


Figure 1.6: Symbol Recognition Palette

### To use the Symbol Recognition palette:

1. With your mouse, draw a symbol in the handwriting recognition region (sketch area).

2. Click the  button. A list of potential matching symbols is displayed. To view more symbols (where indicated), click the drop-down arrows associated with the displayed symbols.

3. To insert a symbol, click the displayed symbol.

## Symbol Names

Each symbol has a name, and some have aliases. By entering its name (or an alias) in Math mode, you can insert the symbol.

**Note:** If you hover the mouse pointer over a palette item, a tooltip displays the symbol's name.

## Using Symbol Names

**To insert a symbol by entering its name:**

1. In Math mode, enter the symbol name.
2. Press the symbol completion shortcut key. See Table 1.3.

Maple inserts the corresponding symbol.

**Table 1.3: Symbol Completion Shortcut Keys**

Operating System	Shortcut Key
Windows	<b>Ctrl + Space</b>
Macintosh	<b>Command + Shift + Space</b>
UNIX	<b>Ctrl + Shift + Space</b>

**For example, to find the square root of 603729 :**

1. Enter *sqrt*.
2. Press the completion shortcut key. Maple displays a pop-up list of exact

matches .

3. In the completion list, select  $\sqrt{x}$ . Maple inserts the symbol with the  $x$  placeholder selected.
4. Enter 603729.
5. Press **Ctrl + =** (**Command + =**, for Macintosh).

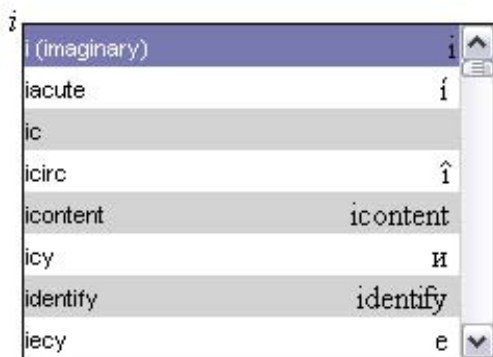
$$\sqrt{603729} = 777$$

## Using Partial Symbol Names

To enter a symbol quickly, you can enter the first few characters of its name and then press the completion shortcut key (see Table 1.3).

- If a unique symbol name matches the characters entered, Maple inserts the corresponding symbol.
- If multiple symbol names match the characters entered, Maple displays the completion list, which lists all matches. To select an item, click its name or symbol.

For example, if you enter  $i$  and then press the completion shortcut key, Maple displays:



### For example, to multiply two complex numbers:

- Use the symbol name and completion list to enter the imaginary unit,


$$i = \sqrt{-1} .$$

$$(-0.123 + 0.745 i) \cdot (4.2 - i) = 0.2284 + 3.2520 i$$

### Example: Indefinite Integral

You can enter any expression using symbol names and the completion list.

For example, to enter the indefinite integral  $\int \sin(x) dx$  :

1. In Math mode, enter **int**. Press the completion shortcut key.
2. From the completion list, select the indefinite integral item .
3. Enter **sin(x)**.
4. Enter **d**. Press the completion shortcut key.
5. From the completion list, select **d (differential)**.
6. Enter **x**.

**Note:** From the **int** completion list, you can directly insert  $\int f dx$ .

## 1.7 Performing Computations

Using the Document mode, you can access the power of the advanced Maple mathematical engine without learning Maple syntax. In addition to solving problems, you can also easily plot expressions.

The primary tools for syntax-free computation are:

- Palettes
- Context menus
- Assistants and tutors

**Note:** The Document mode is designed for quick calculations, but it also supports Maple commands. For information on commands, see *Commands (page 40)* in Chapter 2, *Worksheet Mode (page 35)*.

**Important:** In Document mode, you can execute a statement *only if* you enter it in Math mode. To use a Maple command, you must enter it in Math mode.

## Computing with Palettes

As discussed in *Palettes* (page 10), some palettes contain mathematical operations.

### To perform a computation using a palette mathematical operation:

1. In a palette that contains operators, such as the **Expression** palette, click an operator item.
2. In the inserted item, specify values in the placeholders.
3. To execute the operation and display the result, press **Ctrl+=** (**Command+=**, for Macintosh) or **Enter**.

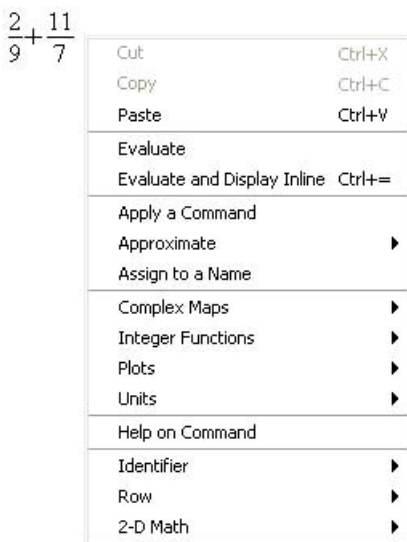
For example, to evaluate  $\int_0^1 e^{-x^2} dx$  inline:

1. Using the **Expression** palette, enter the definite integral. See *Table 1.2* (page 12).
2. Press **Ctrl+=** (**Command+=**, for Macintosh).

$$\int_0^1 e^{-x^2} dx = \frac{1}{2} \operatorname{erf}(1) \sqrt{\pi}$$

## Context Menus

A *context menu* is a pop-up menu that lists the operations and actions you can perform on a particular expression. See Figure 1.7.



**Figure 1.7: Context Menu**

**To display the context menu for an expression:**

- Right-click (**Control**-click, for Macintosh) the expression.

The context menu is displayed beside the mouse pointer.

You can evaluate expressions using context menus.

- The **Evaluate and Display Inline** operation (see Figure 1.7) is equivalent to pressing **Ctrl+=** (**Command+=**, for Macintosh). That is, it inserts an equal sign (=) and then the value of the expression.
- The **Evaluate** operation (see Figure 1.7) is equivalent to pressing **Enter**. That is, it evaluates the expression and displays the result centered on the following line.

For more information on evaluation, see *Evaluating Expressions* (page 8).

From the context menu, you can also select operations different from evaluation. To the right of the expression, Maple inserts a right arrow symbol ( $\rightarrow$ ) and then the result.

For example, use the **Approximate** operation to approximate a fraction:

$$\frac{2}{3} \rightarrow 0.6666666667$$

You can perform a sequence of operations by repeatedly using context menus.

For example, to compute the second derivative of  $\cos(x^2)$ , use the **Differentiate** operation on the expression, and then again on the output:

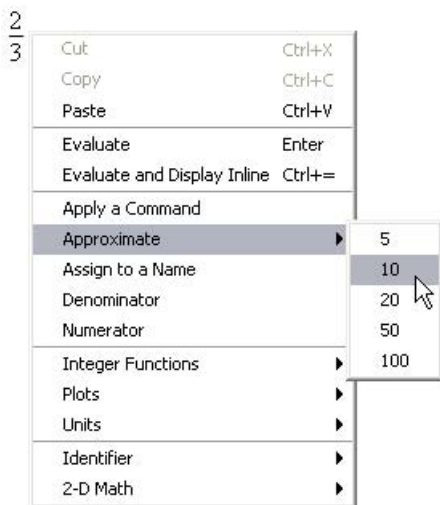
$$\cos(x^2) \rightarrow -2 \sin(x^2) x \rightarrow -4 \cos(x^2) x^2 - 2 \sin(x^2)$$

The following subsections provide detailed instructions on performing a few of the numerous operations available using context menus. Figures in the subsections show related context menus or palettes.

## Approximating the Value of an Expression

### To approximate a fraction numerically:

1. Enter a fraction.
2. Display the context menu. See Figure 1.8.
3. From the context menu, select **Approximate**, and then the number of significant digits to use: **5**, **10**, **20**, **50**, or **100**.



**Figure 1.8: Approximating the Value of a Fraction**

$$\frac{2}{3} \rightarrow 0.6666666667$$

You can replace the inserted right arrow with text or mathematical content.

**To replace the right arrow (  $\rightarrow$  ):**

1. Select the arrow. Press **Delete**.
2. Enter the replacement text or mathematical content.

**Note:** To replace the the right arrow with text, you must first press **F5** to switch to Text mode.

For example, you can replace the arrow with the text "is approximately equal to" or the symbol  $\approx$ .

$$\frac{2}{3} \text{ is approximately equal to } 0.6666666667$$



$$\frac{2}{3} \approx 0.6666666667$$

## Solving an Equation

You can find an exact (*symbolic*) solution or an approximate (*numeric*) solution of an equation. For more information on symbolic and numeric computations, see *Symbolic and Numeric Computation* (page 66).

### To solve an equation:

1. Enter an equation.
2. Display the context menu.
3. From the context menu, select **Solve** or **Solve Numerically**.

$$\frac{7x^2}{3} - x = 12 \rightarrow \left\{ x = \frac{3}{14} + \frac{3}{14}\sqrt{113} \right\}, \left\{ x = \frac{3}{14} - \frac{3}{14}\sqrt{113} \right\}$$

$$\frac{7x^2}{3} - x = 12 \rightarrow \{x = -2.063602674\}, \{x = 2.492174103\}$$

For more information on solving equations, including solving inequations, differential equations, and other types of equations, see *Solving Equations* (page 78).

## Using Units

You can create expressions with units. To specify a unit for an expression, use the **Units** palettes. The **Units (FPS)** palette (Figure 1.9) contains important units from the foot-pound-second (FPS) system of units used in the United States. The **Units (SI)** palette (Figure 1.10) contains important units from the international system (SI) of units.

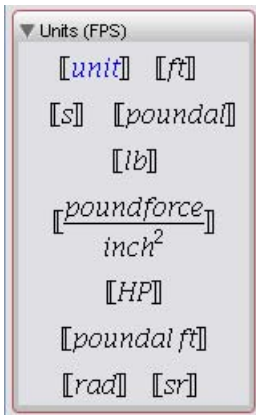


Figure 1.9: FPS Units Palette



Figure 1.10: SI Units Palette

**To insert an expression with a unit:**

1. Enter the expression.
2. In a unit palette, click a unit symbol.

**Note:** To include a reciprocal unit, divide by the unit.

**To evaluate an expression that contains units:**

1. Enter the expression using the units palettes to insert units.
2. Right-click (**Control**-click, for Macintosh) the expression.
3. From the context menu, select **Units** and then **Simplify**.

For example, compute the electric current passing through a wire that conducts 590 coulombs in 2.9 seconds.

$$\frac{590[C]}{2.9[s]} \rightarrow 203.4482759[A]$$

For more information on using units, see *Units* (page 97).

## Assistants and Tutors

Assistants and tutors provide point-and-click interfaces, with buttons, text input regions, and sliders.

### Assistants

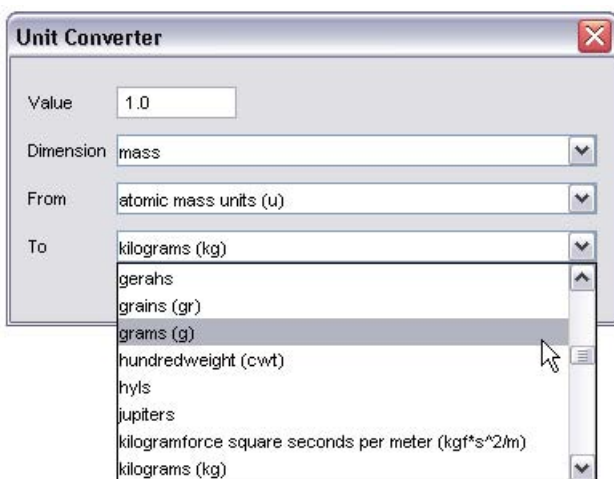
Assistants help you accomplish many tasks, such as solving ordinary differential equations (ODEs) and ODE systems, creating plots and matrices, curve fitting, and performing unit conversions (Figure 1.11).

- From the **Tools** menu, select **Assistants**, and then one of the topic sub-menus.

For example:

To convert a quantity of 1 atomic mass unit to the equivalent value in grams, use the **Unit Convert Assistant**.

1. From the **Tools** menu, select **Unit Converter**. The **Unit Converter** dialog displays. See Figure 1.11.
2. In the **Value** text field, enter the value as measured in atomic mass units, **1.0**.
3. In the **Dimension** drop-down list, select **mass**.
4. In the **From** drop-down list, select **atomic mass units (u)**.
5. In the **To** drop-down list, select **grams (g)**.
6. Click the **Insert** button. Maple inserts the value of the mass in grams,  $.1660540200 \cdot 10^{-23}$ , in the document. **Note:** The output has no units.



**Figure 1.11: Unit Converter Assistant**

## Tutors

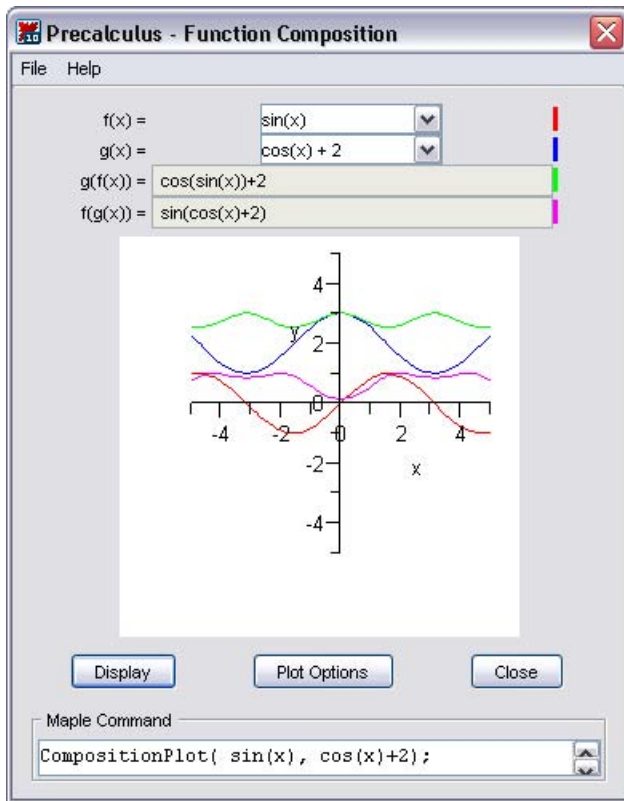
Over 40 interactive tutors help student users gain insight and understanding of topics in courses such as precalculus, calculus, multivariate calculus, vector calculus, and linear algebra. Some tutors help you work through a problem step-by-step.

- From the **Tools** menu, select **Tutors**, and then one of the topic submenus.

For example, you can create a plot of the compositions of two functions using the **Function Composition Tutor**.

### To use the Function Composition Tutor:

1. From the **Tools** menu, select **Tutors**, **Precalculus**, and then **Compositions**. The **Function Composition Tutor** is displayed. See Figure 1.12.
2. In the  $f(x) =$  and  $g(x) =$  text fields, enter the two functions.
3. Click the **Display** button. The tutor displays the compositions  $g(f(x))$  and  $f(g(x))$ .
4. To insert the plot into your document, click the **Close** button.



**Figure 1.12: Function Composition Tutor**

## Using a Context Menu to Launch the Plot Builder

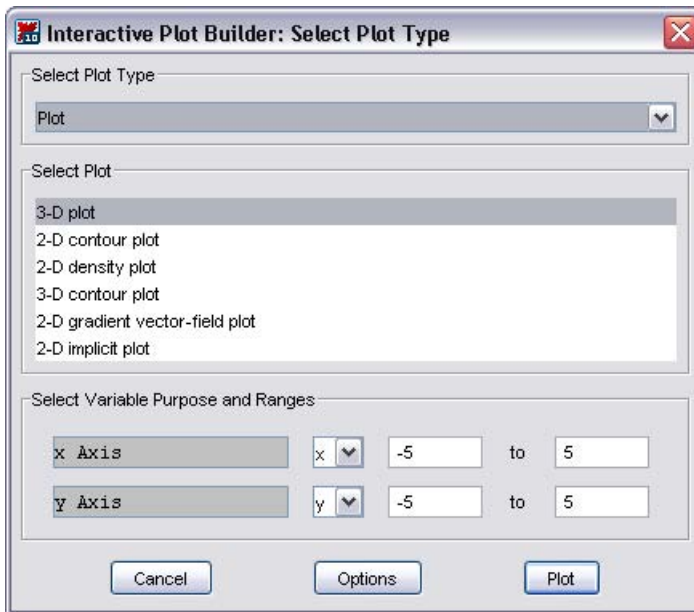
You can plot a mathematical expression using the **Interactive Plot Builder**.

The **Plot Builder** can be launched from the **Tools** menu or the context menu for an expression.

### To create a plot using a context menu:

1. Enter or compute a mathematical expression with one or two independent variables.
2. Right-click (**Control**-click, for Macintosh) the expression to plot.

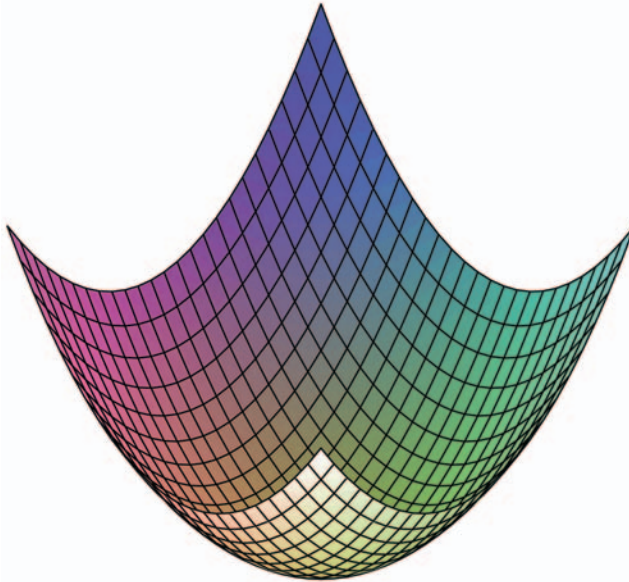
3. From the context menu, select **Plots**, and then **Plot Builder**. The **Interactive Plot Builder** is displayed. See Figure 1.13.
4. In the **Select Plot Type** dialog, select the plot type, for example, **3-D plot** or **2-D contour plot**.
5. To immediately create a plot, click the **Plot** button. To customize the plot before generating it, click the **Options** button.



**Figure 1.13: Interactive Plot Builder: Select Plot Type Dialog**

For example, Figure 1.14 shows a plot of  $x^2 + y^2$ .

$$x^2 + y^2 \rightarrow$$





**Figure 1.14: 3-D Plot of an Expression**

For more information on plots, see *Plots and Animations* (page 189).

## 1.8 Document Mode Summary

The key features of Document mode are summarized in Table 1.4.

**Table 1.4: Summary of Document Mode Tools**

Action	Methods
<p><b>Entering Mathematical Expressions</b></p> <p>For example:</p> $\sin\left(\frac{\pi}{3}\right) + i\cos\left(\frac{\pi}{4}\right)$ $\int_0^{1.9} x^2 \sin(x^2 + 1) \, dx$	<ul style="list-style-type: none"> <li>• Math editing shortcut keys, including symbol name completion</li> <li>• Palettes</li> </ul>
<p><b>Evaluating Mathematical Expressions (Result Inline)*</b></p> <p>For example:</p> $\sin\left(\frac{\pi}{3}\right) + i\cos\left(\frac{\pi}{4}\right) = \frac{1}{2}\sqrt{3} + \frac{1}{2}i\sqrt{2}$	<ul style="list-style-type: none"> <li>• <b>Ctrl += (Command +=, for Macintosh)</b></li> <li>• From the context menu, select <b>Evaluate and Display Inline</b>.</li> </ul>
<p><b>Evaluating Mathematical Expressions (Result Centered on Following Line)</b></p> <p>For example:</p> $\int_0^{1.9} x^2 \sin(x^2 + 1) \, dx$ <p style="text-align: center;">-0.1213460375</p>	<ul style="list-style-type: none"> <li>• <b>Enter</b> key</li> <li>• From the context menu, select <b>Evaluate</b>.</li> </ul>
<p><b>Performing Computations and Other Operations on Expressions</b></p> <p>For example, differentiate an expression:</p> $\sin(x^2 + 1) \rightarrow 2 \cos(x^2 + 1)x$	<ul style="list-style-type: none"> <li>• Context menus</li> <li>• Assistants</li> <li>• Tutors</li> </ul>
<p><b>Executing a Group of Evaluations, Computations, or Other Operations</b></p>	<ul style="list-style-type: none"> <li>• Execute toolbar icon </li> <li>• Execute All toolbar icon </li> </ul>



\* Inline evaluation is available in Document mode and document blocks. For information on document blocks, see *Document Blocks* (page 247).

## 1.9 Getting Help

The Maple Help System contains resources to help you use Maple. See Table 1.5.

**Table 1.5: Maple Help Resources**

Resource	Description
Maple Tour	An interactive overview of Maple. <ul style="list-style-type: none"> <li>From the <b>Help</b> menu, select <b>Take a Tour of Maple</b>.</li> </ul>
Online Manuals	Online manuals, including the <i>Maple Getting Started Guide</i> and this manual. You can execute examples, copy content into other documents, and search the contents using the Maple Help System. <p>The <i>Maple Getting Started Guide</i> provides extensive information for new users on using Maple and the resources available on the Maplesoft Web site (<a href="http://www.maplesoft.com">http://www.maplesoft.com</a>). Each procedure and concept is accompanied by visual depictions of actions to help you identify Maple resources and tools.</p> <ul style="list-style-type: none"> <li>From the <b>Help</b> menu, select <b>Manuals, Dictionary, and more&gt;Manuals</b>.</li> </ul>
Quick Help	A list of key commands and concepts. <ul style="list-style-type: none"> <li>From the <b>Help</b> menu, select <b>Quick Help</b>. Alternatively, press <b>F1</b>. For additional information, click an item in the Quick Help.</li> </ul>
Quick Reference	A table of commands and information for new users that opens in a new window. It contains hyperlinks to help pages for more information. <ul style="list-style-type: none"> <li>From the <b>Help</b> menu, select <b>Quick Reference</b>. Alternatively, press <b>Ctrl + F2</b> (<b>Command + F2</b>, for Macintosh).</li> </ul>

Resource	Description
Help Pages	<p>Help for Maple features, commands, packages, and more. Help pages include examples and screenshots to help you quickly learn.</p> <ul style="list-style-type: none"> <li>From the <b>Help</b> menu, select <b>Maple Help</b>. You can search for a help topic, perform a text search, or browse the <b>Table of Contents</b>. You can also open a help page by entering <b>?&lt;topic_name&gt;</b> at the input prompt (in Worksheet mode) or in Math mode (in Document mode).</li> </ul>
Task Templates	<p>Set of commands with placeholders that you can use to quickly perform a task.</p> <ul style="list-style-type: none"> <li>From the <b>Tools</b> menu, select <b>Tasks</b>, and then <b>Browse</b>.</li> </ul>
Applications and Example Worksheets	<p>Executable documents that demonstrate the power and flexibility of the Maple interactive document or provide an overview of computations in a particular field.</p> <ul style="list-style-type: none"> <li>From the <b>Help</b> menu, select <b>Manuals, Dictionary, and more</b>, and then <b>Applications and Examples</b>.</li> </ul>
Mathematics and Engineering Dictionary	<p>Over 5000 definitions, including 300 figures and plots.</p> <ul style="list-style-type: none"> <li>From the <b>Help</b> menu, select <b>Manuals, Dictionary, and more</b>, and then <b>Dictionary</b>.</li> </ul>

For a complete list of resources, refer to the **?MapleResources** help page.

For more information on the Maple Help System, refer to the *Maple Getting Started Guide*.



## 2 Worksheet Mode

The *Worksheet* mode of the Standard Worksheet interface is designed for:

- Interactive use through Maple commands, which may offer advanced functionality or customized control not available using context menus or other syntax-free methods
- Programming using the powerful Maple language

Using Worksheet mode, you have access to most of the Maple features described in Chapter 1 including:

- Math and Text modes
- Palettes
- Context menus
- Assistants and tutors

For information on these features, see Chapter 1, *Document Mode (page 1)*. (For a summary, see *Table 1.4 (page 30)*.)

**Note:** Using a document block, you can use all Document mode features in Worksheet mode. For information on document blocks, see *Document Blocks (page 247)*.

**Note:** This chapter and the following chapters were created using Worksheet mode.

## 2.1 In This Chapter

Section	Topics
Input Prompt - Where you enter input	<ul style="list-style-type: none"> <li>• The Input Prompt (&gt;)</li> <li>• Suppressing Output</li> <li>• 2-D and 1-D Math Input</li> <li>• Input Separators</li> </ul>
Commands - Thousands of routines for performing computations and other operations	<ul style="list-style-type: none"> <li>• The Maple Library</li> <li>• Top-Level Commands</li> <li>• Package Commands</li> </ul>
Palettes - Items that you can insert by clicking or dragging	<ul style="list-style-type: none"> <li>• Using Palettes</li> </ul>
Context Menus - Pop-up menus of common operations	<ul style="list-style-type: none"> <li>• Using Context Menus</li> </ul>
Assistants and Tutors - Graphical interfaces with buttons and sliders	<ul style="list-style-type: none"> <li>• Launching Assistants and Tutors</li> <li>• Example: Using the Interactive Plot Builder</li> </ul>
Task Templates - Sets of commands with placeholders that you can insert and use to perform a task	<ul style="list-style-type: none"> <li>• Viewing Task Templates</li> <li>• Inserting a Task Template</li> <li>• Performing the Task</li> </ul>
Text Regions - Areas in the document in which you can enter text	<ul style="list-style-type: none"> <li>• Inserting a Text Region</li> <li>• Formatting Text</li> </ul>
Names - References to the expressions you assign to them	<ul style="list-style-type: none"> <li>• Assigning to Names</li> <li>• Unassigning Names</li> <li>• Valid Names</li> </ul>

Section	Topics
Equation Labels - Automatically generated labels that you can use to refer to expressions	<ul style="list-style-type: none"> <li>• Displaying Equation Labels</li> <li>• Referring to a Previous Result</li> <li>• Execution Groups with Multiple Outputs</li> <li>• Label Numbering Schemes</li> <li>• Features of Equation Labels</li> </ul>

## 2.2 Input Prompt

In Worksheet mode, you enter input at the Maple *input prompt* (>). The default mode for input is Math mode (*2-D Math*).

To evaluate input:

- Press **Enter**.

Maple displays the result (output) below the input.

For example, to find the value of  $\sin^3(\pi/3)$ , enter the expression, and then press **Enter**.

$$> \sin^3\left(\frac{\pi}{3}\right)$$

$$\frac{3}{8}\sqrt{3}$$

For example, compute the sum of two fractions.

$$> \frac{2}{9} + \frac{7}{11}$$

$$\frac{85}{99}$$

## Suppressing Output

To suppress the output, enter a colon (: ) at the end of the input.

$$> \frac{2}{9} + \frac{7}{11} :$$

A set of Maple input and its output are referred to as an *execution group*.

## 1-D Math Input

You can also insert input using Text mode (*1-D Math*). The input is entered as a one-dimensional sequence of characters. 1-D Math input is red.

### To enter input using 1-D Math:

- At the input prompt, press **F5** to switch from 2-D Math to 1-D Math.

$$> 123^2 - 29857/120;$$

$$\frac{1785623}{120}$$

**Important:** 1-D Math input must end with a semicolon or colon. If you use a semicolon, Maple displays the output. If you use a colon, Maple suppresses the output.

$$> 123^2 - 29857/120:$$

### To set the default input mode to 1-D Math:

1. From the **Tools** menu, select **Options**. The **Options** dialog is displayed.

2. On the **Display** tab, in the **Input display** drop-down list, select **Maple Notation**.
3. Click **Apply to Session** (to set for only the current session) or **Apply Globally** (to set for all Maple sessions).

### To convert 2-D Math input to 1-D Math input:

1. Select the 2-D Math input.
2. From the **Format** menu, select **Convert To**, and then **1-D Math Input**.

**Important:** In Document mode, you can execute a statement *only if* you enter it in Math mode.

## Input Separators

In 1-D and 2-D Math input, you can use a semicolon or colon to separate multiple inputs in the same input line.

>  $\sqrt{4.4}; \tan(3.2)$

2.097617696

0.05847385446

If you do not specify a semicolon or colon, Maple interprets it as a single input.

>  $\sqrt{4.4} \tan(3.2)$

0.1226557919



## 2.3 Commands

Maple contains a large set of commands and a powerful programming language. Most Maple commands are written using the Maple programming language.

You can enter commands using 1-D or 2-D Math. You must use 1-D Math input when programming in Maple. *Basic Programming (page 321)* provides an introduction to Maple programming.

To learn how to use Maple commands, use task templates. See *Task Templates (page 51)*.

### The Maple Library

Commands are contained in the Maple library, which is divided into two groups: the *top-level commands* and *packages*.

- The top-level commands are the most frequently used Maple commands.
- Packages contain related specialized commands in areas such as student calculus, linear algebra, vector calculus, and code generation.

For a complete list of packages and commands, refer to the **index** help pages. To access the index overview help page, enter **?index**, and then press **Enter**. For information on the Maple Help System, see *Getting Help (page 32)*.

### Top-Level Commands

To use a top-level command, enter its name followed by parentheses (( )) containing any parameters. This is referred to as a *calling sequence* for the command.

<code>command(arguments)</code>
---------------------------------

**Note:** In 1-D Math input, include a semicolon or colon at the end of the calling sequence.

For example, to differentiate an expression, use the **diff** command. The required parameters are the expression to differentiate, which must be specified first, and the independent variable.

> *diff*(tan(x) sin(x), x)

$$(1 + \tan(x)^2) \sin(x) + \tan(x) \cos(x)$$

For a complete list of functions (commands that implement mathematical functions), for example, **BesselI** and **AiryAi**, available in the library, refer to the **?initialfunctions** help page. (To display this help page, enter **?initialfunctions** at the input prompt.)

>  $\frac{\text{BesselI}(0.1, 1)}{\text{AiryAi}(2.2)}$

47.53037086

For detailed information on the properties of a function, use the **FunctionAdvisor** command.

> *FunctionAdvisor*('definition', BesselI)

$$\left[ I_a(z) = \frac{z^a {}_0F_1\left( ; 1+a; \frac{z^2}{4} \right)}{\Gamma(1+a) 2^a}, \text{ with no restrictions on } (a, z) \right]$$

For detailed information on how to use a function in Maple, refer to its help page.

For example:

> *?Bessel*

**Note:** In 1-D and 2-D Math input, when accessing a help page using **?**, you do not need to include a trailing semicolon or colon.

## Package Commands

To use a package command, the calling sequence must include the package name, and the command name enclosed in brackets ([ ]).

```
package[command](arguments)
```

If you are frequently using the commands in a package, load the package.

### To load a package:

- Use the **with** command, specifying the package as an argument.

The **with** command returns a list of the package commands loaded (unless you suppress the output by entering a colon at the end of the calling sequence).

After loading a package, you can use its commands as top-level commands, that is, without specifying the package name.

For example, use the **NLPSolve** command from the **Optimization** package to find a *local* minimum of an expression and the value of the independent variable at which the minimum occurs.

```
> Optimization[NLPSolve]( $\frac{\sin(x)}{x}, x = 1..15$ )  
[-0.0913252028230576718, [x = 10.9041216700744900]]
```

```
> with(Optimization):
```

```
> NLPSolve( $\frac{\sin(x)}{x}, x = 1..15$ )  
[-0.0913252028230576718, [x = 10.9041216700744900]]
```

For more information on optimization, see *Optimization* (page 168).

**To unload a package:**

- Use the **unwith** command, specifying the package as an argument.

> *unwith(Optimization):*

To use the examples in this manual, you may be required to use the **unwith** command between examples.

Some packages contain commands that have the same name as a top-level command. When you load one of these packages, Maple returns a warning.

For example, the **plots** package contains a **changecoords** command. Maple also contains a top-level **changecoords** command.

> *with(plots):*

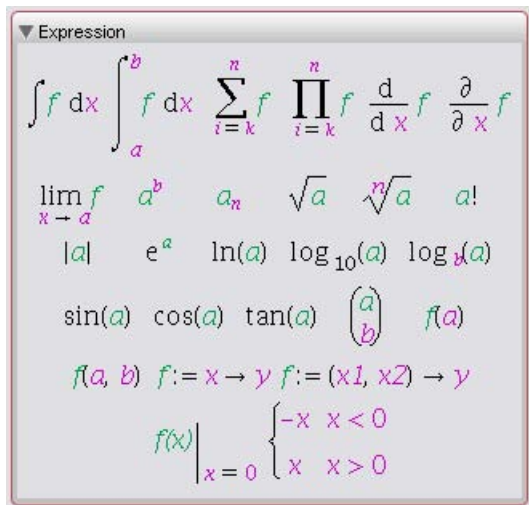
Warning, the name **changecoords** has been redefined

In general, this manual does not include the warning messages Maple returns.

To use the top-level command, unload the package. (For alternative methods of accessing the top-level command, refer the **?with** help page.)

## 2.4 Palettes

Palettes are collections of related items that you can insert by clicking or dragging. See Figure 2.1.



**Figure 2.1: Expression Palette**

You can use palettes to enter input.

For example, evaluate a definite integral using the definite integration item


$$\int_a^b f dx$$

in the **Expression** palette.

In 2-D Math, clicking the definite integration item inserts:

$$> \int_a^b f dx$$

1. Enter values in the placeholders. To move to the next placeholder, press

**Tab. Note:** If pressing the **Tab** key inserts a tab, click the Tab icon  in the toolbar.

2. To evaluate the integral, press **Enter**.

$$\begin{aligned} > \int_0^1 \tanh(x) \, dx \\ & \qquad \qquad \qquad - \ln(2) + \ln(e^{-1} + e) \end{aligned}$$

In 1-D Math, clicking the definite integration item inserts the corresponding command calling sequence.

> **int(f,x=a..b);**

Specify the problem values (using the **Tab** to move to the next placeholder), and then press **Enter**.

> **int(tanh(x), x = 0..1):**

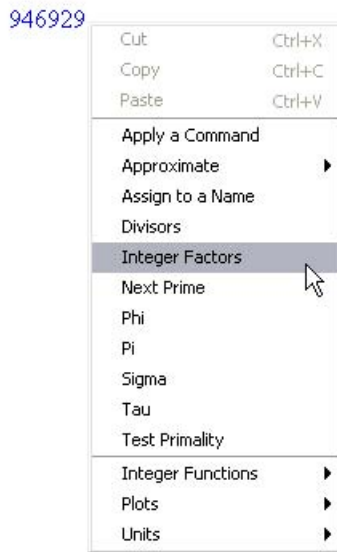
**Note:** Some palette items cannot be inserted into 1-D Math because they are not defined in the Maple language. When the cursor is in 1-D Math input, unavailable palette items are dimmed.

For more information on palettes, see *Palettes (page 10)* and *Performing Computations (page 19)* in Chapter 1.

## 2.5 Context Menus

A *context menu* is a pop-up menu that lists the operations and actions you can perform on a particular expression. See Figure 2.2.

> 946929



**Figure 2.2: Integer Context Menu**

In Worksheet mode, you can use context menus to perform operations on 2-D Math and output.

### To use a context menu:

1. Right-click (**Control**-click, for Macintosh) the expression. The context menu is displayed.
2. From the context menu, select an operation.

Maple inserts a new execution group containing:

- The calling sequence that performs the operation
- The result of the operation

For example:

**To determine a rational expression (fraction) that approximates a floating-point number:**

1. Right-click (**Control-click**, for Macintosh) the floating-point number.
2. From the context menu, select **Conversions**, and then **Rational**.

The inserted calling sequence includes an equation label reference to the number you are converting.

```
> 0.3463678 + 1.7643  
  
2.1106678 (2.1)
```

```
> convert( (2.1), 'rational' )  
  

$$\frac{32270}{15289}$$

```

For information on equation labels and equation label references, see *Equation Labels* (page 59).

For more information on context menus, see *Context Menus* (page 20) in Chapter 1.



## 2.6 Assistants and Tutors

Assistants and tutors provide point-and-click interfaces, with buttons, text input regions, and sliders. See Figure 2.3.

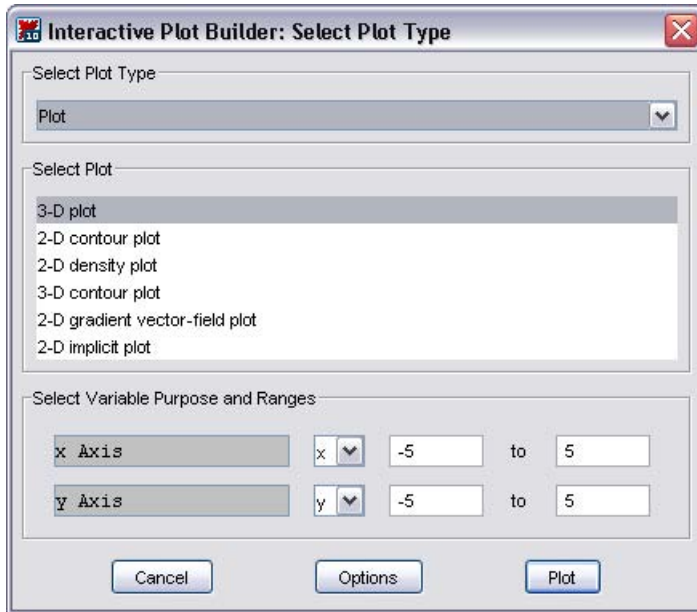


Figure 2.3: Interactive Plot Builder: Select Plot Type Dialog

### Launching an Assistant or Tutor

**To launch an assistant or tutor:**

1. Open the **Tools** menu.
2. Select **Assistants** or **Tutors**.
3. Navigate to and select one of the assistants or tutors.

## Example: Using the Interactive Plot Builder

To plot an expression using the Interactive Plot Builder:

1. From the **Tools** menu, select **Assistants**, and then **Plot Builder**. Maple inserts the following command in the document and launches the **Interactive Plot Builder**.

```
> plots[interactive]();
```

2. In the **Interactive Plot Builder: Specify Expressions** window (Figure 2.4), click **Add**. The **Add/Edit Expression** dialog is displayed.

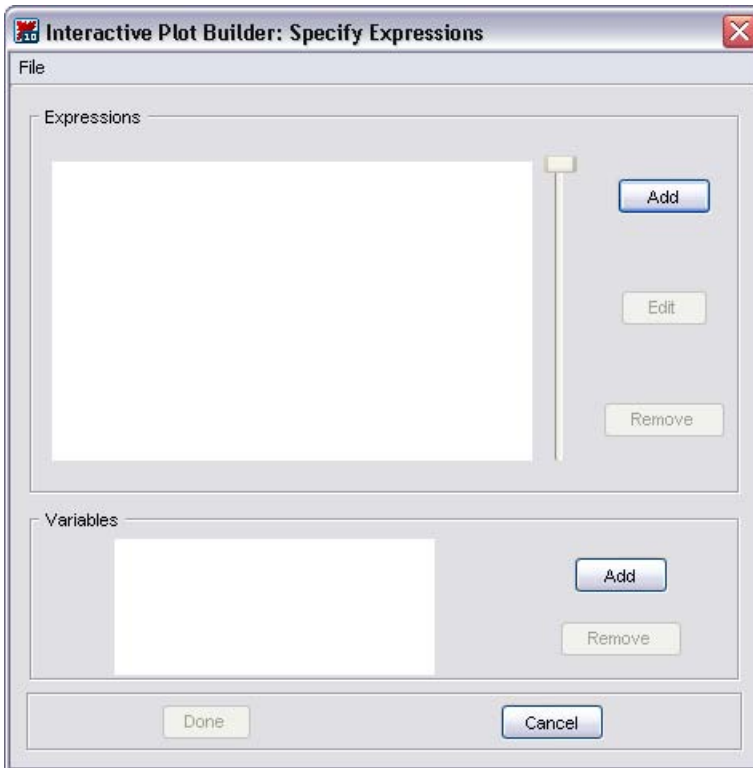
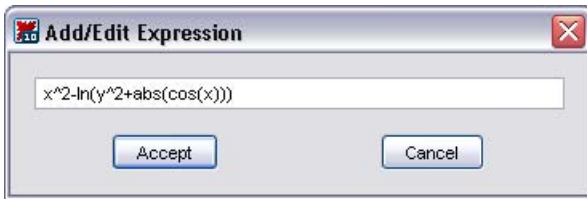


Figure 2.4: Interactive Plot Builder: Specify Expressions Window

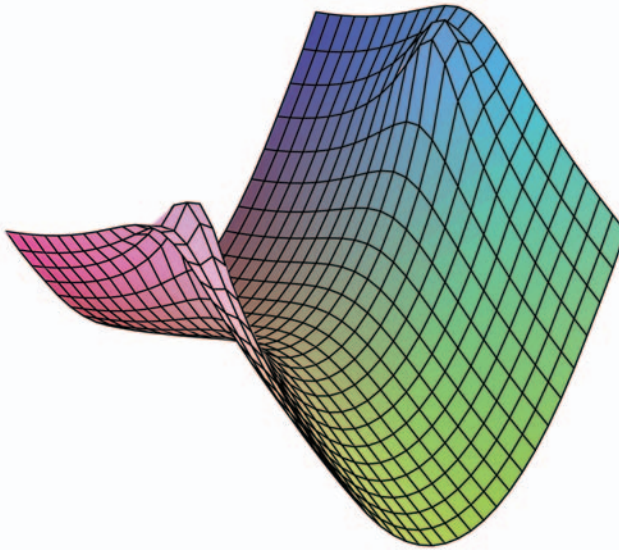
3. In the **Add/Edit Expression** dialog, enter the expression to plot using 1-D Math. See Figure 2.5.



**Figure 2.5: Interactive Plot Builder: Add/Edit Expression Dialog**

4. Repeat steps 2 and 3 for each expression to add to the plot.
5. After adding the expressions, in the **Interactive Plot Builder: Specify Expressions** window (Figure 2.4), click **Done**.
6. In the **Interactive Plot Builder: Select Plot Type** dialog (Figure 2.3), select the type of plot, for example, **3-D Plot** or **3-D Contour Plot**, and specify the variable ranges, for example,  $x = [-2, 2], y = [-2, 2]$ .
7. To immediately create a plot, click **Plot**. To customize the plot before generating it, click **Options**. Set the plot options, and then click **Plot**.

Maple inserts the plot in the document.



For more information on assistants and tutors, see *Assistants and Tutors* (page 26) in Chapter 1.

## 2.7 Task Templates

Maple can solve a diverse set of problems. The task template facility helps you quickly find and use the commands required to perform common tasks.

After inserting a task template, specify the parameters of your problem in the placeholders, and then execute the commands, or click a button.

### Viewing Task Templates

The **Task Browser** (Figure 2.6) organizes task templates by subject.

**To launch the Task Browser:**

- From the **Tools** menu, select **Tasks**, and then **Browse**.

You can also browse the task templates in the Table of Contents of the Maple Help System.

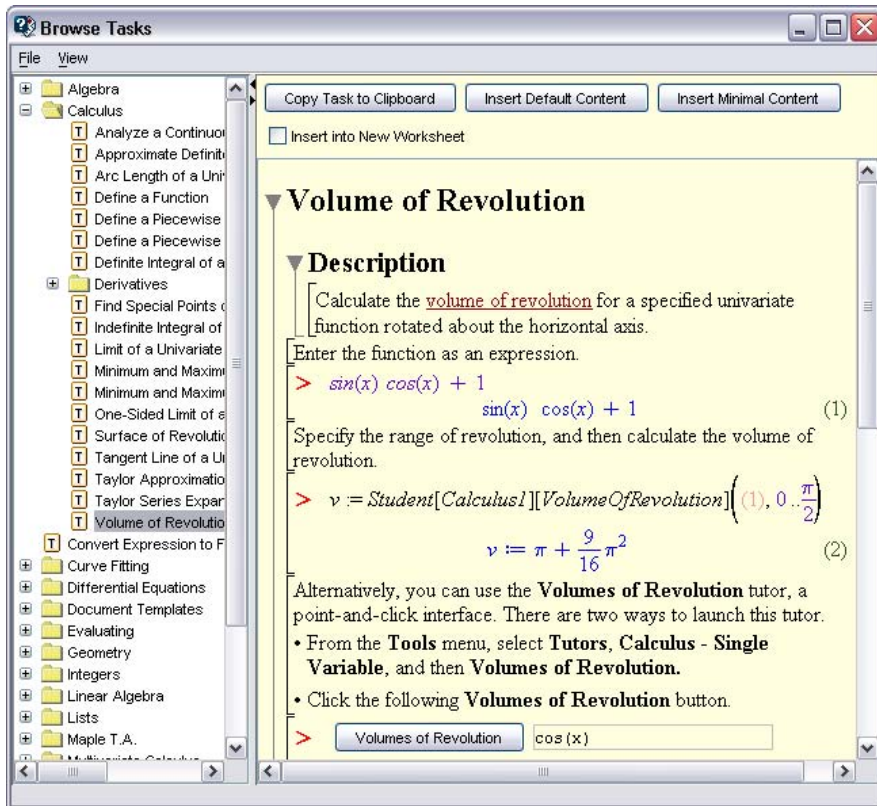


Figure 2.6: Task Browser

## Inserting a Task Template

To insert a task template from the Task Browser or Help Navigator:

1. Navigate to the task.
2. Click one of the insertion or copy buttons.

- Click the **Insert Default Content** button. Maple inserts the *default content*. The default content level is set using the **Options** dialog. For details, see the following steps.
- Click the **Insert Minimal Content** button. Maple inserts only the commands and embedded components, for example, a button to launch the related assistant or tutor.
- Click the **Copy Task to Clipboard** button. Place the cursor where you want to insert the task, and then paste the task. Maple inserts the default content. Use this method to quickly insert a task multiple times.

**To change the default content level, use the Options dialog.**

1. From the **Tools** menu, select **Options**. The **Options** dialog opens.
2. In the **Options** dialog, select the **Display** tab.
3. In the **Task content to insert** drop-down list, select **All Content**, **Standard Content**, or **Minimal Content**.

- Minimal Content - Only the commands and embedded components
- Standard Content - Commands, embedded components, and instructions for using the template
- All Content - All content in the task template, including hyperlinks to related help pages

Maple stores a list of the most recently inserted task templates.

**To insert a recently inserted task:**


- From the **Tools** menu, select **Tasks**, and then the task name.

Maple inserts the default content.

## Performing the Task

After you insert a task template, enter the parameters for your task, and then compute the result.

**To use an inserted task template:**


1. Specify values for the parameters in placeholders or using graphical interface components. You can move to the next placeholder by pressing **Tab**.
2. Execute all commands in the task by:
  - Placing the cursor in the first task command, and then pressing **Enter** repeatedly to execute each command.
  - Selecting all the template commands, and then clicking the execute toolbar icon .
3. If the template contains a button that computes the result, click it.

For more information on task templates, refer to the *Maple Getting Started Guide* or the **?tasks** help page.

## 2.8 Text Regions

To add descriptive text in Worksheet mode, use a *text region*.

**To insert a text region:**

- In the toolbar, click the Text region icon .

The default mode in a text region is Text mode.

In a text region, you can:

- Enter text with inline mathematical content by switching between Text and Math modes. To toggle between Text mode and Math mode, press **F5**. **Note:** The mathematical content in a text region is not evaluated. To enter mathematical content that is evaluated, enter it at an *Input Prompt* (page 37).
- Insert any palette item. Palette items are inserted in Math mode (2-D Math).

You can format text in a text region. Features include:

- Character styles
- Paragraph styles
- Sections and subsections
- Tables

For more information on formatting documents, see *Creating Mathematical Documents* (page 231).

## 2.9 Names

Instead of re-entering an expression every time you need it, you can assign it to a *name* or add an *equation label* to it. Then you can quickly refer to the expression using the name or an equation label reference. For information on labels, see the following section *Equation Labels* (page 59).

### Assigning to Names

You can assign any Maple expression to a name: numeric values, data structures, procedures (a type of Maple program), and other Maple objects.

Initially, the value of a name is itself.

```
> a
```

*a*

The assignment operator ( $:=$ ) associates an expression with a name.

```
> a := π
```

*a := π*



Recall that you can enter  $\pi$  using the following two methods.

- Use the **Common Symbols** palette
- In 2-D Math enter  $pi$ , and then press the symbol completion short cut key. See *Shortcuts for Entering Mathematical Expressions* (page 6).

When Maple evaluates an expression that contains a name, it replaces the name with its value. For example:

```
> cos(a)
```

-1

For information on Maple evaluation rules, see *Evaluating Expressions* (page 310).

## Mathematical Functions

To define a function, assign it to a name.

For example, define a function that computes the cube of its argument.

```
> cube := x → x3:
```

For information on creating functions, see *Defining a Mathematical Function* (page 14).

```
> cube(3); cube(1.666)
```

27

4.624076296

**Note:** To insert the right arrow, enter the characters `->`. In 2-D Math, Maple replaces `->` with the right arrow symbol  $\rightarrow$ . In 1-D Math, the characters are not replaced.

For example, define a function that squares its argument.

```
> square := x -> x^2:
```

```
> square(32);
```

1024

For more information on functions, see *Functional Operators* (page 292).

## Protected Names

Protected names are valid names that are predefined or reserved.

If you attempt to assign to a protected name, Maple returns an error.

```
> sin := 2
```

Error, attempting to assign to `sin` which is protected

For more information, refer to the **?type/protected** and **?protect** help pages.

## Unassigning Names

The **unassign** command resets the value of a name to itself. **Note:** You must enclose the name in right single quotes ('').

```
> unassign('a')
```

```
> a
```

*a*

Right single quotes (*unevaluation quotes*) prevent Maple from evaluating the name. For more information on unevaluation quotes, see *Delaying Evaluation* (page 317) or refer to the **?uneval** help page.

See also *Unassigning a Name Using Unevaluation Quotes* (page 319).

## Unassigning All Names

The **restart** command clears the Maple internal memory. The effects include unassigning all names and unloading all packages. For more information, refer to the **?restart** help page.

**Note:** To use the examples in this manual, you may be required to use the **unassign** or **restart** command between examples.

## Valid Names

A Maple name must be one of the following.

- A sequence of alphanumeric and underscore (`_`) characters that begins with an alphabetical character. **Note:** To enter an underscore character in 2-D Math, enter a backslash character followed by an underscore character, that is, `\_`.
- A sequence of characters enclosed in left single quotes (```).

**Important:** Do not begin a name with an underscore character. Maple reserves names that begin with an underscore for use by the Maple library.

Examples of valid names:

- **a**
- **a1**
- **polynomial**
- **polynomial1\_divided\_by\_polynomial2**
- ``2a``
- ``x#y``
- ``x y``

## 2.10 Equation Labels

Maple marks the output of each execution group with a unique equation label.

**Note:** The equation label is displayed to the right of the output.

$$\begin{aligned} > \int \sin(x) \, dx \\ & \qquad \qquad \qquad - \cos(x) \qquad \qquad \qquad (2.2) \end{aligned}$$

Using equation labels, you can refer to the result in other computations.

$$\begin{aligned} > \int (2.2) \, dx \\ & \qquad \qquad \qquad - \sin(x) \qquad \qquad \qquad (2.3) \end{aligned}$$

### Displaying Equation Labels

**Important:** By default, equation labels are displayed. If equation label display is turned off, complete **both** the following operations.

- From the **Format** menu, select **Labels**, and then ensure that **Worksheet** is selected.
- In the **Options** dialog (**Tools>Options**), on the **Display** tab, ensure that **Show equation labels** is selected.

### Referring to a Previous Result

Instead of re-entering previous results in computations, you can use equation label references. Each time you need to refer to a previous result, insert an equation label reference.


**To insert an equation label reference:**

- From the **Insert** menu, select **Label**. (Alternatively, press **Ctrl+L**. For Macintosh, **Command+L**.)
- In the **Insert Label** dialog, enter the label value, and then click **OK**.

Maple inserts the reference.

For example:

**To integrate the product of (2.2) and (2.3):**

1. In the **Expression** palette, click the indefinite integration item . The item is inserted and the cursor moves to the integrand placeholder.
2. Press **Ctrl+L** (**Command+L**, for Macintosh).
3. In the **Insert Label** dialog, enter **2.2**. Click **OK**.
4. Press **\***.
5. Press **Ctrl+L** (**Command+L**, for Macintosh).
6. In the **Insert Label** dialog, enter **2.3**. Click **OK**.
7. To move to the variable of integration placeholder, press **Tab**.
8. Enter **x**. Press **Enter**.

$$> \int (2.2) \cdot (2.3) \, dx$$

$$\frac{1}{2} \sin(x)^2$$

## Execution Groups with Multiple Outputs

An equation label is associated with the *last output* within an execution group.

$$> \left(\frac{2}{3.5}\right)^2; \cos\left(\frac{\pi}{6}\right)$$

0.3265306122

$$\frac{1}{2}\sqrt{3}$$

(2.4)

$$> (2.4)^2$$

$$\frac{3}{4}$$

## Label Numbering Schemes

You can number equation labels in two ways:

- **Flat** - Each label is a single number, for example, 1, 2, or 3.
- **Sections** - Each label is numbered according to the section in which it occurs. For example, 2.1 is the first equation in the second section, and 1.3.2 is the second equation in the third subsection of the first section.

**To change the equation label numbering scheme:**

- From the **Format** menu, select **Labels**, and then **Label Display**. In the **Format Labels** dialog (Figure 2.7), select one of the formats.
- Optionally, enter a prefix.

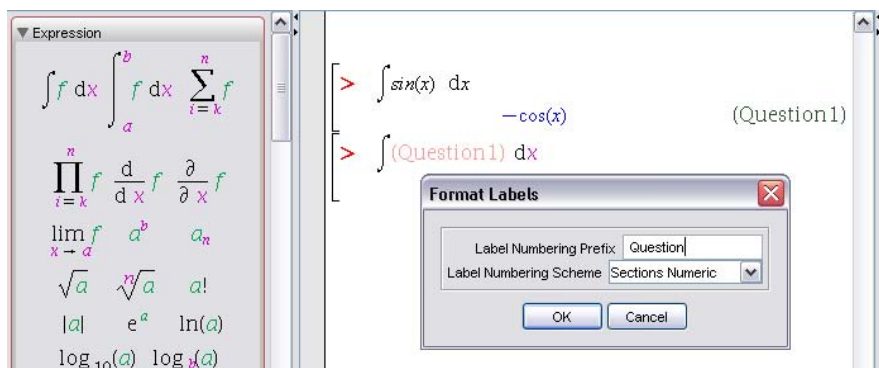


Figure 2.7: Format Labels Dialog: Adding a Prefix

## Features of Equation Labels

Although equation labels are not descriptive names, labels offer other important features.

- Each label is unique, whereas a name may be inadvertently assigned to more than once for different purposes.
- Maple labels the output values sequentially. If you remove or insert an output, Maple automatically renumbers all equation labels and updates the label references.
- If you change the equation label format (see *Label Numbering Schemes (page 61)*), Maple automatically updates all equation labels and label references.

For information on assigning to, using, and unassigning names, see *Names (page 55)*.

For more information on equation labels, refer to the **?equationlabels** help page.

The following chapters describe how to use Maple to perform tasks such as solving differential (and other types of) equations, producing plots and animations, and creating mathematical documents. The chapters were created

using Worksheet mode. Except where noted, all features are available in both Worksheet mode and Document mode.





# 3 Performing Computations

This chapter discusses key concepts related to performing computations with Maple. It discusses important features that are relevant to all Maple users. After learning about these concepts, you will learn how to use Maple to solve problems in specific areas in the following chapter.

## 3.1 In This Chapter

Section	Topics
Symbolic and Numeric Computation - An overview of exact and floating-point computation	<ul style="list-style-type: none"><li>• Exact Computations</li><li>• Floating-Point Computations</li><li>• Converting Exact Quantities to Floating-Point Values</li><li>• Sources of Error</li></ul>
Integer Operations - How to perform integer computations	<ul style="list-style-type: none"><li>• Important Integer Commands</li><li>• Non-Base 10 Numbers</li><li>• Finite Rings and Fields</li><li>• Gaussian Integers</li></ul>
Solving - How to solve standard mathematical equations	<ul style="list-style-type: none"><li>• Equations and Inequations</li><li>• Ordinary Differential Equations</li><li>• Partial Differential Equations</li><li>• Integer Equations</li><li>• Integer Equations in a Finite Field</li><li>• Linear Systems</li><li>• Recurrence Relations</li></ul>

Section	Topics
Units, Scientific Constants, and Uncertainty - How to construct and compute with expressions that have units, scientific constants, or uncertainty	Units <ul style="list-style-type: none"> <li>• Conversions</li> <li>• Applying Units to an Expression</li> <li>• Performing Computations with Units</li> <li>• Changing the Current System of Units</li> <li>• Extensibility</li> </ul> Scientific Constants <ul style="list-style-type: none"> <li>• Scientific Constants</li> <li>• Element and Isotope Properties</li> <li>• Value, Units, and Uncertainty</li> <li>• Performing Computations</li> <li>• Modification and Extensibility</li> </ul> Uncertainty Propagation <ul style="list-style-type: none"> <li>• Quantities with Uncertainty</li> <li>• Performing Computations with Quantities with Uncertainty</li> </ul>
Restricting the Domain - How to restrict the domain for computations	<ul style="list-style-type: none"> <li>• Real Number Domain</li> <li>• Assumptions on Variables</li> </ul>

## 3.2 Symbolic and Numeric Computation

*Symbolic computation* is the mathematical manipulation of expressions involving symbolic or abstract quantities, such as variables, functions, and operators; and exact numbers, such as integers, rationals,  $\pi$ , and  $e^2$ . The goal of such manipulations may be to transform an expression to a simpler form or to relate the expression to other, better understood formulas.

*Numeric computation* is the manipulation of expressions in the context of finite-precision arithmetic. Expressions involving exact numbers, for example,  $\sqrt{2}$ , are replaced by close approximations using floating-point numbers, for example 1.41421. These computations generally involve some error. Understanding and controlling this error is often of as much importance as the computed result.

In Maple, numeric computation is normally performed if you use floating-point numbers (numbers containing a decimal point) or the **evalf** command. The **plot** command (see *Plots and Animations (page 189)*) uses numeric computation, while commands such as **int**, **limit**, and **gcd** (see *Integer Operations (page 71)* and *Mathematical Computations (page 123)*) generally use only symbolic computation to achieve their results.

## Exact Computations

In Maple, integers, rational numbers, mathematical constants such as  $\pi$  and  $\infty$ , and mathematical structures such as matrices with these as entries are treated as exact quantities. Names, such as **x**, **y**, **my\_variable**, and mathematical functions, such as  $\sin(x)$  and  $LambertW(k, z)$ , are *symbolic* objects. Names can be assigned exact quantities as their values, and functions can be evaluated at symbolic or exact arguments.

$$> \frac{3}{2} + \frac{1}{3}, 1 + \frac{\pi}{2}$$

$$\frac{11}{6}, 1 + \frac{1}{2}\pi$$

**Important:** Unless requested to do otherwise (see the following section), Maple evaluates expressions containing exact quantities to exact results, as you would do if you were performing the calculation by hand, and not to numeric approximations, as you normally obtain from a standard hand-held calculator.

>  $\sin(1), \sin(\pi), \sin(x)$

$\sin(1), 0, \sin(x)$

>  $\int \tan(t) \, dt$

$-\ln(\cos(t))$

>  $\sqrt{32}$

$4\sqrt{2}$

## Floating-Point Computations

In some situations, a numeric approximation of an exact quantity is required. For example, the **plot** command requires the expression it is plotting to evaluate to numeric values that can be rendered on the screen:  $\pi$  cannot be so rendered, but 3.14159 can be. Maple distinguishes *approximate* from *exact* quantities by the presence or absence of a decimal point: 1.9 is approximate, while  $\frac{19}{10}$  is exact.

**Note:** An alternative representation of floating-point numbers, called *e-notation*, may not include an explicit decimal point:  $1e5 = 100000.$ ,  $3e-2 = .03$ .

In the presence of a floating-point (approximate) quantity in an expression, Maple generally computes using numeric approximations. Arithmetic involving mixed exact and floating-point quantities results in a floating-point result.

>  $1.5 + \frac{2}{3}$

2.166666667

If a mathematical function is passed a floating-point argument, it normally attempts to produce a floating-point approximation to the result.

$$> \sin(1.5), \int_{0.0}^{1.0} e^x dx$$

0.9974949866, 1.718281828

## Converting Exact Quantities to Floating-Point Values

To convert an exact quantity to a numeric approximation of that quantity, use the **evalf** command or the **Approximate** context menu operation (see *Approximating the Value of an Expression* (page 22)).

$$> \text{evalf}(\pi), \text{evalf}(\sin(3)), \text{evalf}\left(\frac{3}{2} + \frac{1}{3}\right)$$

3.141592654, 0.1411200081, 1.833333333

By default, Maple computes such approximations using 10 digit arithmetic. You can modify this in one of two ways:

- *Locally*, you can pass the precision as an index to the **evalf** call.

$$> \text{evalf}[20](\exp(2)), \text{evalf}\left(\Gamma\left(\frac{2}{3}\right)\right)$$

7.3890560989306502272, 1.354117939

- *Globally*, you can set the value of the **Digits** environment variable.

$$> \text{Digits} := 5;$$

```
> evalf(tan(pi/3))
```

1.7321

For more information, see the **?evalf** and **?Digits** help pages.

**Note:** When appropriate, Maple performs floating-point computations directly using your computer's underlying hardware.

## Sources of Error

By its nature, floating-point computation normally involves some error. Controlling the effect of this error is the subject of active research in *Numerical Analysis*. Some sources of error are:

- An exact quantity may not be exactly representable in decimal form:  $\frac{1}{3}$  and  $\pi$  are examples.
- Small errors can accumulate after many arithmetic operations.
- Subtraction of nearly equal quantities can result in essentially no useful information. For example, consider the computation  $x - \sin(x)$  for  $x \approx 0$ .

```
> (x-sin(x))|_{x=.00001}
```

0.

No correct digits remain. If, however, you use Maple to analyze this expression, and replace this form with a representation that is more accurate for small values of  $x$ , a fully accurate 10-digit result can be obtained.

```
> t:=taylor(x-sin(x),x)
```

$$t := \frac{1}{6}x^3 - \frac{1}{120}x^5 + O(x^6)$$

```
> t |
   |x=.00001
```

```
.1666666667 10-15
```

For information on evaluating an expression at a point, see *Substituting a Value for a Subexpression* (page 310). For information on creating a series approximation, see *Series* (page 161).

For more information on floating-point numbers, refer to the **?float** and **?type/float** help pages.

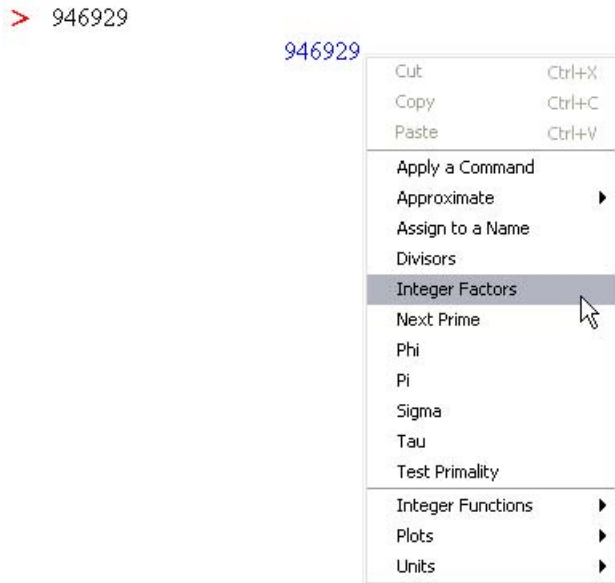
## 3.3 Integer Operations

In addition to the basic arithmetic operators, Maple has many specialized commands for performing more complicated integer computations, such as factoring an integer, testing whether an integer is a prime number, and determining the greatest common divisor (GCD) of a pair of integers.

**Note:** Many integer operations are available as task templates (**Tools>Tasks>Browse**).

You can quickly perform many integer operations using context menus. Selecting an integer, and then right-clicking (for Macintosh, **Control**-clicking) displays a context menu with integer commands, for example, **Integer Factors**, which applies the **ifactor** command. See Figure 3.1.





**Figure 3.1: Context Menu for an Integer**

In Worksheet mode, Maple uses an equation label reference in the **ifactor** calling sequence.

> 9469629

$$9469629 \tag{3.1}$$

> `ifactor((3.1))`

$$(3)^4 (13) (17) (23)^2$$

For more information on equation labels, see *Equation Labels* (page 59).

For more information on using context menus in Worksheet mode, see *Context Menus* (page 46). For information on using context menus in Document mode, see *Context Menus* (page 20).

You can also enter the **ifactor** command and specify the integer to factor as an argument.

```
> ifactor(9469629)
```

$$(3)^4 (13) (17) (23)^2$$

Maple has many other integer commands, including those listed in Table 3.1.

**Table 3.1: Select Integer Commands**

Command	Description
<b>abs</b>	absolute value (displays in 2-D math as $ a $ )
<b>factorial</b>	factorial (displays in 2-D math as $a!$ )
<b>ifactor</b>	factorization
<b>igcd</b>	greatest common divisor
<b>iquo</b>	quotient of integer division
<b>irem</b>	remainder of integer division
<b>iroot</b>	integer approximation of nth root
<b>isprime</b>	test primality
<b>isqrt</b>	integer approximation of square root
<b>max, min</b>	maximum and minimum of a set
<b>mod</b>	modular arithmetic (See <i>Finite Rings and Fields</i> (page 75))
<b>numtheory[divisors]</b>	set of positive divisors

```
> iquo(209, 17)
```

> *irem*(209, 17)

5

> *igcd*(2024, 4862)

22

> *iroot*(982523, 4)

31

For information on finding integer solutions to equations, see *Integer Equations* (page 94).

## Non-Base 10 Numbers and Other Number Systems

Maple supports:

- Non-base 10 numbers
- Finite ring and field arithmetic
- Gaussian integers

### Non-Base 10 Numbers

To represent an expression in another base, use the **convert** command.

> *convert*(6000, 'binary')

1011101110000

> *convert*(34271, 'hex')

85DF

For information on enclosing keywords in right single quotes (`'`), see *Delaying Evaluation* (page 317).

You can also use the **convert/base** command.

```
> convert(34271, 'base', 16)
```

```
[15, 13, 5, 8]
```

**Note:** The **convert/base** command returns a list of digit values in order of *increasing significance*.

### Finite Rings and Fields

Maple supports computations over the integers modulo  $m$ .

The **mod** operator evaluates an expression over the integers modulo  $m$ .

```
> 27 mod 4
```

```
3
```

By default, the **mod** operator uses positive representation (**modp** command). Symmetric representation is available using the **mods** command.

```
> modp(27, 4)
```

```
3
```

```
> mods(27, 4)
```

```
-1
```

For information on setting symmetric representation as the default, refer to the **?mod** help page.

The modular arithmetic operators are listed in Table 3.2.

**Table 3.2: Modular Arithmetic Operators**

Operation	Operator	Example
Addition	+	> $7 + 6 \bmod 5$ 3
Subtraction	-	> $\text{mods}(3 - 16, 11)$ -2
Multiplication (displays in 2-D Math as $\cdot$ )	*	> $13 \cdot 5 \bmod 3$ 2
Multiplicative inverse (displays in 2-D Math as a superscript)	<b>^-1</b>	> $3^{(-1)} \bmod 5$ 2
Division (displays in 2-D Math as $\frac{a}{b}$ )	/	> $\frac{2}{3} \bmod 5$ 4
Exponentiation <sup>1</sup>	<b>&amp;^</b>	> $(100 \&^ 100) \bmod 7$ 2

<sup>1</sup>To enter a caret (^) in 2-D Math, enter a backslash character followed by a caret, that is, \^.

For information on solving an equation modulo an integer, see *Integer Equations in a Finite Field* (page 95).

The **mod** operator also supports polynomial and matrix arithmetic over finite rings and fields. For more information, refer to the **?mod** help page.

## Gaussian Integers

Gaussian integers are complex numbers in which the real and imaginary parts are integers.

The **GaussInt** package contains commands that perform Gaussian integer operations.

The **GIfactor** command returns the Gaussian integer factorization.

```
> GaussInt[GIfactor](173 + 16 i)
      (1 + 2 i)(41 - 66 i)
```

You can enter the imaginary unit using the following two methods.

- In the **Common Symbols** palette, click the **i** or **j** item. See *Palettes (page 10)*.
- Enter *i* or *j*, and then press the symbol completion key. See *Symbol Names (page 16)*.

**Note:** In 1-D Math input, enter the imaginary unit as an uppercase **i** (**I**).

The **GIsqrt** command approximates the square root in the Gaussian integers.

```
> GaussInt[GIsqrt](9 - 5 j)
      3 - i
```

For more information on Gaussian integers including a list of **GaussInt** package commands, refer to the **?GaussInt** help page.

## 3.4 Solving Equations

You can solve a variety of equation types, including those described in Table 3.3.

**Table 3.3: Overview of Solution Methods for Important Equation Types**

Equation Type	Solution Method
Equations and inequations	<b>solve</b> and <b>fsolve</b> commands
Ordinary differential equations	<b>ODE Analyzer Assistant</b> (and <b>dsolve</b> command)
Partial differential equations	<b>pdsolve</b> command
Integers equations	<b>isolve</b> command
Integer equations in a finite field	<b>msolve</b> command
Linear systems	<b>LinearAlgebra[LinearSolve]</b> command
Recurrence relations	<b>rsolve</b> command

**Note:** Many solve operations are available as task templates (**Tools>Tasks>Browse**) and in context menus. This section focuses on other methods.

### Solving Equations and Inequations

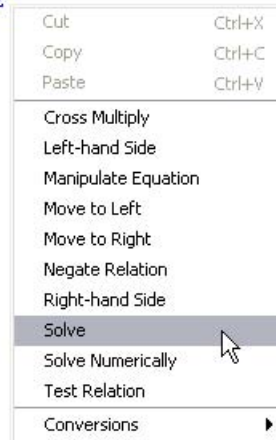
Using Maple, you can symbolically solve equations and inequations. You can also solve equations numerically.

**To solve an equation or set of equations using context menus:**

1. Right-click (for Macintosh, **Control**-click) the equations.
2. From the context menu, select **Solve** (or **Solve Numerically**). See Figure 3.2.

$$> \frac{7x^2}{3} - x = 12$$

$$\frac{7}{3}x^2 - x = 12$$



**Figure 3.2: Context Menu for an Equation**

In Worksheet mode, Maple inserts a calling sequence that solves the equation followed by the solutions.

If you select **Solve**, Maple computes exact solutions.

$$> \frac{7x^2}{3} - x = 12$$

$$\frac{7}{3}x^2 - x = 12 \tag{3.2}$$

> `solve( {(3.2)} )`

$$\left\{ x = \frac{3}{14} + \frac{3\sqrt{113}}{14} \right\}, \left\{ x = \frac{3}{14} - \frac{3\sqrt{113}}{14} \right\}$$

If you select **Solve Numerically**, Maple computes floating-point solutions.



$$> \frac{7x^2}{3} - x = 12$$

$$\frac{7}{3}x^2 - x = 12 \quad (3.3)$$

> `fsolve( {(3.3)} )`

`{x = -2.063602674}, {x = 2.492174103}`

For information on solving equations and inequations symbolically using the **solve** command, see the following section. For information on solving equations numerically using the **fsolve** command, see *Numerically Solving Equations* (page 84).

### Symbolically Solving Equations and Inequations

The **solve** command is a general solver that determines exact symbolic solutions to equations or inequations. The solutions to a single equation or inequation are returned as an expression sequence. If Maple does not find any solutions, the **solve** command returns the empty expression sequence.

> `solve(x2 + x = 256)`

$$-\frac{1}{2} + \frac{5}{2}\sqrt{41}, -\frac{1}{2} - \frac{5}{2}\sqrt{41}$$

It is recommended that you verify the solutions returned by the **solve** command. For details, see *Working with Solutions* (page 86).

To return the solutions as a list, enclose the calling sequence in brackets ([ ]).

> `[solve(x2 + x = 256, x)]`

$$\left[ -\frac{1}{2} + \frac{1}{2}\sqrt{1 + 1024y}, -\frac{1}{2} - \frac{1}{2}\sqrt{1 + 1024y} \right]$$

**Expressions** You can specify expressions instead of equations. The **solve** command automatically equates them to zero.

$$> \text{solve}(e^z + z)$$

$$- W(1)$$

$W$  represents the Lambert  $W$  function.

**Multiple Equations** To solve multiple equations or inequations, specify them as a set or list.

$$> \text{solve}([xy^2 - y = 5, x > 0])$$

$$\left\{ x = \frac{y+5}{y^2}, -5 < y, y < 0 \right\}, \left\{ x = \frac{y+5}{y^2}, 0 < y \right\}$$

$$> \text{solve}([xy^2 - y = 5, x < 0])$$

$$\left\{ x = \frac{y+5}{y^2}, y < -5 \right\}$$

**Solving for Specific Unknowns** By default, the **solve** command returns solutions for all unknowns. You can specify the unknowns for which to solve.

$$> \text{solve}(q^2 - rs + \frac{q}{r} = 5, q)$$

$$\frac{1}{2} \frac{-1 + \sqrt{1 + 4r^3s + 20r^2}}{r}, -\frac{1}{2} \frac{1 + \sqrt{1 + 4r^3s + 20r^2}}{r}$$

To solve for multiple unknowns, specify them as a **list**.

$$\begin{aligned} > \text{solve}\left(\left\{\frac{q}{s} - \frac{r}{s+1} + \frac{q}{r} = 5, rs = 1\right\}, [q, r]\right) \\ & \left\{q = \frac{1 + 5s^2 + 5s}{s + 1 + s^3 + s^2}, r = \frac{1}{s}\right\} \end{aligned}$$

**Transcendental Equations** In general, the **solve** command returns one solution to transcendental equations.

$$> \text{equation1} := \sin(x) = \cos(x):$$

$$> \text{solve}(\text{equation1})$$

$$\frac{1}{4}\pi$$

To produce all solutions, set the **\_EnvAllSolutions** environment variable to **true**.

**Note:** To enter an underscore character (`_`) in 2-D Math, enter `\_`.

$$> \_EnvAllSolutions := true :$$

$$> \text{solve}(\text{equation1})$$

$$\frac{1}{4}\pi + \pi\_Z1\sim$$

Maple uses variables of the form `\_ZN\sim`, where **N** is a positive integer, to represent arbitrary integers. The tilde (`~`) indicates that it is a quantity with an assumption. For information about names with assumptions, see *Assumptions on Variables* (page 117).

**RootOf Structure** The **solve** command may return solutions, for example, to higher order polynomial equations, in an implicit form using **RootOf** structures.

```
> [solve(x5 - 2x4 + 3x3 - 2)]
```

$$\left[ 1, \text{RootOf}\left(\_Z^4 - \_Z^3 + 2\_Z^2 + 2\_Z + 2, \text{index}=1\right), \right. \\ \left. \text{RootOf}\left(\_Z^4 - \_Z^3 + 2\_Z^2 + 2\_Z + 2, \text{index}=2\right), \right. \\ \left. \text{RootOf}\left(\_Z^4 - \_Z^3 + 2\_Z^2 + 2\_Z + 2, \text{index}=3\right), \right. \\ \left. \text{RootOf}\left(\_Z^4 - \_Z^3 + 2\_Z^2 + 2\_Z + 2, \text{index}=4\right) \right] \quad (3.4)$$

These **RootOf** structures are placeholders for the roots of the equation  $z^4 - z^3 + 2z^2 + 2z + 2$ . The **index** parameter numbers and orders the four solutions.

Like any symbolic expression, you can convert **RootOf** structures to a floating-point value using the **evalf** command.

```
> evalf(3.4)
```

$$\left[ 1., 0.9840010519 + 1.526590834 i, \right. \\ \left. -0.4840010519 + 0.6099471405 i, \right. \\ \left. -0.4840010519 - 0.6099471405 i, \right. \\ \left. 0.9840010519 - 1.526590834 i \right]$$

Some equations are difficult to solve symbolically. For example, polynomial equations of order five and greater do not in general have a solution in terms of radicals. If the **solve** command does not find any solutions, it is recommended that you use the Maple numerical solver, **fsolve**. For information, see the following section, *Numerically Solving Equations*.

For more information on the **solve** command, including how to solve equations defined as procedures and how to find parametric solutions, refer to the **?solve/details** help page.

For information on verifying and using solutions returned by the **solve** command, see *Working with Solutions* (page 86).

### Numerically Solving Equations

The **fsolve** command solves equations numerically. The behavior of the **fsolve** command is similar to that of the **solve** command.

```
> equation2 := z cos(z) = 2:
```

```
> fsolve(equation2, z)
```

23.64662473 (3.5)

**Note:** You can also numerically solve equations using the context menus. See *Solving Equations and Inequalities* (page 78).

It is recommended that you verify the solutions returned by the **fsolve** command. For details, see *Working with Solutions* (page 86).

**Multiple Equations** To solve multiple equations, specify them as a set. The **fsolve** command solves for all unknowns.

```
> fsolve({ln(x) = y2 + 1, x y = ey})
```

{x = 3.396618823, y = 0.4719962637}

**Univariate Polynomial Equations** In general, the **fsolve** command finds one solution. However, for a univariate polynomial equation, the **fsolve** command returns all *real* roots.

```
> equation3 := y4 - 3y2 - 2y + 1:
```

```
> fsolve(equation3, y)
```

0.3365322739, 1.940392664

**Controlling the Number of Solutions** To limit the number of roots returned, specify the **maxsols** option.

```
> fsolve(equation3, y, 'maxsols' = 1)
```

0.3365322739

To find additional solutions to a general equation, use the **avoid** option to ignore known solutions.

```
> fsolve(equation2, z, 'avoid' = {z = (3.5)})
```

-2.498755763

**Complex Solutions** To search for a complex solution, or find all complex and real roots for a univariate polynomial, specify the **complex** option.

```
> fsolve(equation3, y, 'complex')
```

-1.138462469 - 0.4850624941 i,  
-1.138462469 + 0.4850624941 i, 0.3365322739, 1.940392664

If the **fsolve** command does not find any solutions, it is recommended that you specify a range in which to search for solutions, or specify an initial value.

**Range** To search for a solution in a range, specify the range in the calling sequence. The range can be real or complex.

```
> fsolve(equation2, z, {z = 100..200})
```

149.2390528

The syntax for specifying a region in the complex plane is **lower-left point..upper-right point**.

```
> fsolve(equation3, y, {y = -2 - i..0}, 'complex');
```

$$-1.138462469 - 0.4850624941 i$$

**Initial Values** You can specify a value for each unknown. The **fsolve** command uses these as initial values for the unknowns in the numerical method.

```
> fsolve(equation2, {z=100})  
  
           {z=98.98037599} (3.6)
```

For more information and examples, refer to the **?fsolve/details** help page.

For information on verifying and using solutions returned by the **fsolve** command, see the following section, *Working with Solutions*.

### Working with Solutions

**Verifying** It is recommended that you always verify solutions (that the **solve** and **fsolve** commands return) using the **eval** command.

```
> equation4 := sin(x) = -cos(x):
```

```
> solve(equation4)
```

$$-\frac{1}{4}\pi (3.7)$$

```
> eval(equation4, x=(3.7))
```

$$-\frac{1}{2}\sqrt{2} = -\frac{1}{2}\sqrt{2}$$

```
> equation5 := cos(z) =  $\frac{2}{z}$ :
```

```
> fsolve(equation5)
```

$$-2.498755763 \quad (3.8)$$

> `eval(equation5, {z=(3.8)})`

$$-0.8003983544 = -0.8003983540$$

For more information, see *Substituting a Value for a Subexpression* (page 310).

**Assigning the Value of a Solution to a Variable** To assign the value of a solution to the corresponding variable as an *expression*, use the **assign** command.

For example, consider the numeric solution to **equation2**, `{z=98.98037599}` (3.6), found using the starting value  $z=100$ .

> `assign((3.6))`

> `z`

$$98.98037599$$

**Creating a Function from a Solution** The **assign** command assigns a value as an expression to a name. It does **not** define a function. To convert a solution to a function, use the **unapply** command.

Consider one of the solutions for **q** to the equation  $q^2 - rs + \frac{q}{r} = 5$ .

> `solutions := [solve(q^2 - rs + q/r = 5, q)]:`

> `f := unapply(solutions[1], r, s)`



$$f := (r, s) \mapsto \frac{1}{2} \frac{-1 + \sqrt{1 + 4r^3s + 20r^2}}{r}$$

You can evaluate this function at symbolic or numeric values.

>  $f(x, y)$

$$\frac{1}{2} \frac{-1 + \sqrt{1 + 4x^3y + 20x^2}}{x}$$

>  $f\left(\frac{1}{\sqrt{2}}, 1\right)$

$$\frac{1}{2}\sqrt{2} \left( -1 + \sqrt{11 + \sqrt{2}} \right)$$

>  $f(5.7, 2.1)$

4.032680522

For more information on defining and using functions, see *Functional Operators* (page 292).

## Other Specialized Solvers

In addition to equations and inequations, Maple can solve other equations including:

- Ordinary differential equations (ODEs)
- Partial differential equations (PDEs)
- Integer equations
- Integer equations in a finite field
- Linear systems

- Recurrence relations

## Ordinary Differential Equations (ODEs)

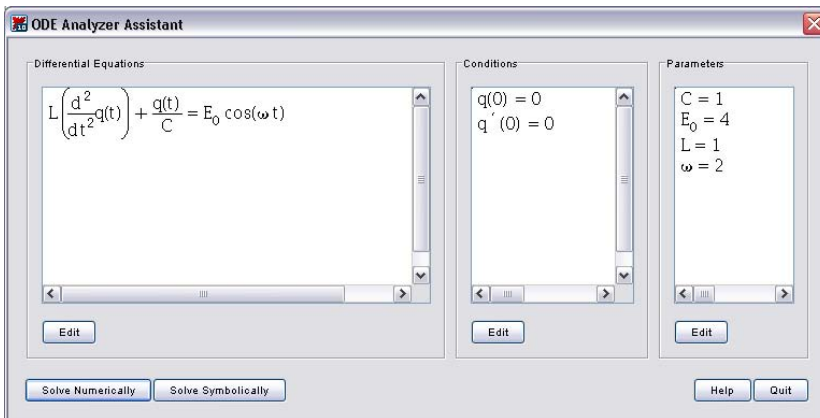
Maple can solve ODEs and ODE systems, including initial value and boundary value problems, symbolically and numerically.

**ODE Analyzer Assistant** The **ODE Analyzer Assistant** is a point-and-click interface to the Maple ODE solving routines.

### To launch the ODE Analyzer:

- From the **Tools** menu, select **Assistants**, and then **ODE Analyzer**.

Maple inserts the `dsolve[interactive]()` calling sequence in the document. The **ODE Analyzer Assistant** (Figure 3.3) is displayed.



**Figure 3.3: ODE Analyzer Assistant**

In the main **ODE Analyzer Assistant** window, you can define ODEs, initial or boundary value conditions, and parameters. To define derivatives, use the

**diff** command. For example, **diff(x(t), t)** corresponds to  $\frac{dx(t)}{dt}$ , and **diff(x(t),**

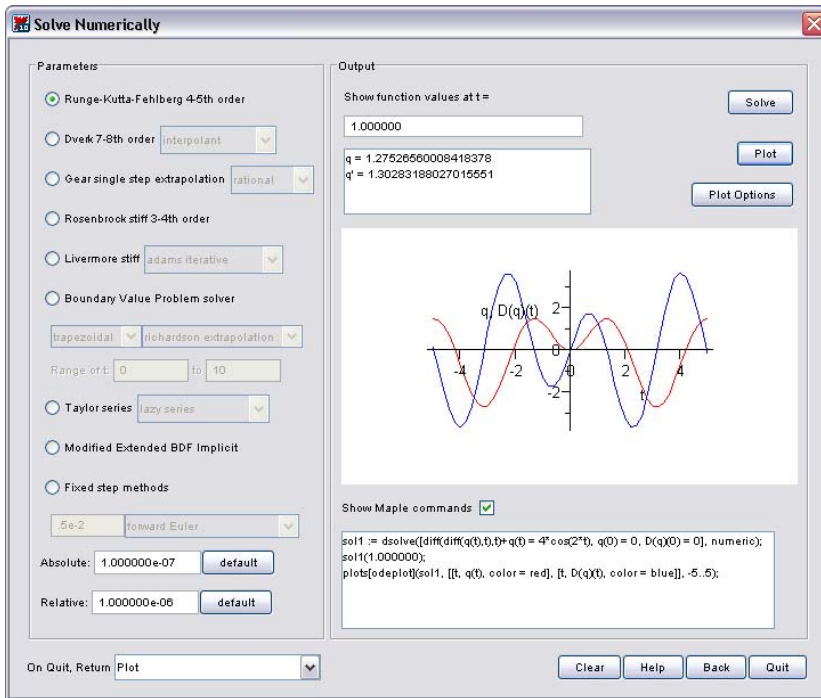
**t, t)** corresponds to  $\frac{d^2x(t)}{dt^2}$ . For more information on the **diff** command, see

*The diff Command (page 157).*

After defining an ODE, you can solve it numerically or symbolically.

**To solve a system numerically using the ODE Analyzer Assistant:**

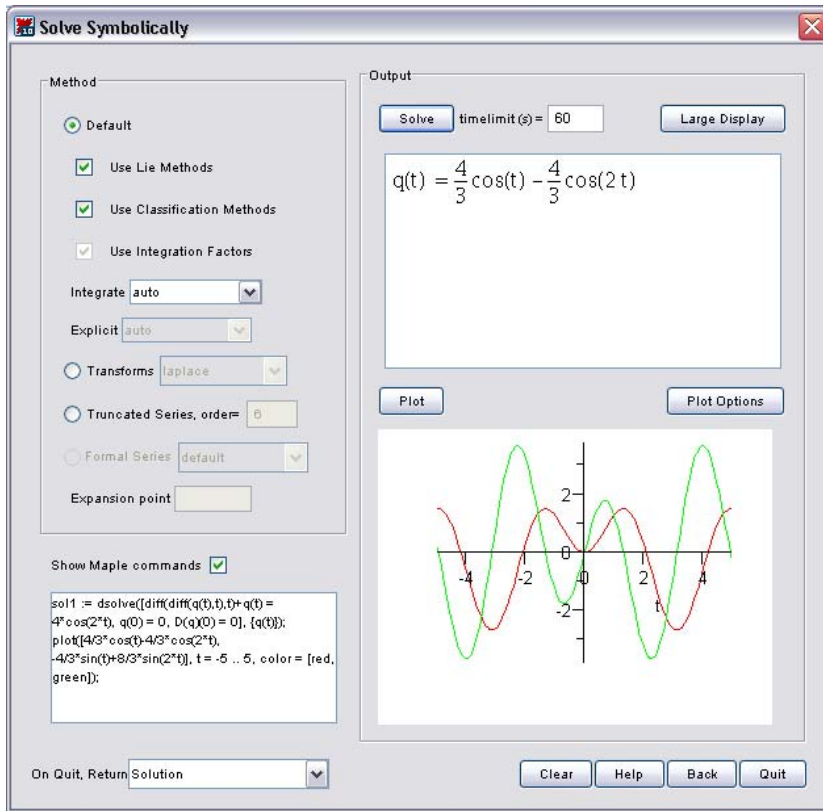
1. Ensure that the conditions guarantee uniqueness of the solution.
2. Ensure that all parameters have fixed values.
3. Click the **Solve Numerically** button.
4. In the **Solve Numerically** window (Figure 3.4), you can specify the numeric method and relevant parameters and error tolerances to use for solving the problem.
5. To compute solution values at a point, click the **Solve** button.



**Figure 3.4: ODE Analyzer Assistant: Solve Numerically Dialog**

### To solve a system symbolically using the ODE Analyzer Assistant:

1. Click the **Solve Symbolically** button.
2. In the **Solve Symbolically** window (Figure 3.5), you can specify the method and relevant method-specific options to use for solving the problem.
3. To compute the solution, click the **Solve** button.



**Figure 3.5: ODE Analyzer Assistant: Solve Symbolically Dialog**

When solving numerically or symbolically, you can view a plot of the solution by clicking the **Plot** button.

- To plot the solution to a symbolic problem, all conditions and parameters must be set.
- To customize the plot, click the **Plot Options** button to open the **Plot Options** window.

To view the corresponding Maple commands as you solve the problem or plot the solution, select the **Show Maple** commands check box.

You can control the return value of the ODE Analyzer using the **On Quit, Return** drop-down list. You can select to return nothing, the displayed plot, the computed numeric procedure (for numeric solutions), the solution (for symbolic solutions), or the Maple commands needed to produce the solution values and the displayed plot.

For more information, refer to the **?ODEAnalyzer** help page.

### The **dsolve** Command

The ODE Analyzer provides a point-and-click interface to the Maple **dsolve** command.

For ODEs or systems of ODEs, the **dsolve** command can find:

- Closed form solutions
- Numerical solutions
- Series solutions

In addition, the **dsolve** command can find:

- Formal power series solutions to linear ODEs with polynomial coefficients
- Formal solutions to linear ODEs with polynomial coefficients

To access all available functionality, use the **dsolve** command directly. For more information, refer to the **?dsolve** help page.

### Partial Differential Equations (PDEs)

To solve a PDE or PDE system symbolically or numerically, use the **pdsolve** command. PDE systems can contain ODEs, algebraic equations, and inequalities.

For example, solve the following PDE symbolically.

$$> x \left( \frac{\partial}{\partial y} f(x, y) \right) - y \left( \frac{\partial}{\partial x} f(x, y) \right) = 0$$

$$x \frac{\partial}{\partial y} f(x, y) - y \frac{\partial}{\partial x} f(x, y) = 0 \quad (3.9)$$

> `pdsolve((3.9))`

$$f(x, y) = \_FI(x^2 + y^2)$$

The solution is an arbitrary univariate function applied to  $x^2 + y^2$ .

Maple generally prints only the return value, errors, and warnings during a computation. To print information about the techniques Maple uses, increase the **infolevel** setting for the command.

To return all information, set **infolevel** to 5.

> `infolevel[pdsolve]:=5:`

> `pdsolve((3.9))`

*Checking arguments ...*

*Getting info and details about the PDE ... First set of solution methods (general or quase general solution) Second set of solution methods (complete solutions) Trying methods for first order PDEs HINT = strip Trying characteristic strip method ... characteristic strip method successful. Second set of solution methods successful*

$$f(x, y) = \_FI(x^2 + y^2)$$

For more information on solving PDEs, including numeric solutions and solving PDE systems, refer to the **?pdsolve** help page.

## Integer Equations

To find only integer solutions to an equation, use the **isolve** command. The **isolve** command finds solutions for all variables. For more information, refer to the **?isolve** help page.

>  $\text{isolve}(\{x^2 + y = 13\})$   
 $\{y = -_Z1^2 + 13, x = _Z1\}$

### Integer Equations in a Finite Field

To solve an equation modulo an integer, use the **msolve** command. For more information, refer to the **?msolve** help page. The **msolve** command finds solutions for all variables.

>  $\text{msolve}(\{x^2 = 1\}, 13)$   
 $\{x = 1\}, \{x = 12\}$

### Solving Linear Systems

To solve a linear system, use the **LinearAlgebra[LinearSolve]** command. For more information, refer to the **?LinearAlgebra[LinearSolve]** help page. The **LinearSolve** command returns the vector **x** that satisfies **A · x = B**.

For example, construct an augmented matrix using the **Matrix** palette (see *Creating Matrices and Vectors (page 135)*) in which the first four columns contain the entries of **A** and the final column contains the entries of **B**.

>  $\text{linearsystem} := \left[ \begin{array}{ccccc} \frac{59}{10} & \frac{44}{25} & \frac{17}{2} & \frac{1}{100} & \frac{1}{2} \\ 1 & 0 & 7 & \frac{533}{100} & \frac{61}{50} \\ 98 & \frac{21}{10} & \frac{3}{10} & 7 & \frac{2178}{25} \\ 23 & 9 & 12 & \frac{51}{10} & \frac{786}{25} \end{array} \right] :$

>  $\text{LinearAlgebra[LinearSolve]}(\text{linearsystem})$



$$\begin{bmatrix} \frac{31753441047}{41858667400} \\ \frac{16991806239}{8371733480} \\ \frac{-1489266217}{1674346696} \\ \frac{262603866}{209293337} \end{bmatrix}$$

For more information on using Maple to solve linear algebra problems, see *Linear Algebra* (page 135).

### Solving Recurrence Relations

To solve a recurrence relation, use the **rsolve** command. For more information, refer to the **?rsolve** help page. The **rsolve** command finds the general term of the function.

> `rsolve({f(n)=f(n-1)+f(n-2),f(0)=1,f(1)=1},{f(n)})`

$$\left\{ f(n) = \left( \frac{1}{2} - \frac{1}{10}\sqrt{5} \right) \left( -\frac{1}{2}\sqrt{5} + \frac{1}{2} \right)^n + \left( \frac{1}{10}\sqrt{5} + \frac{1}{2} \right) \left( \frac{1}{2} + \frac{1}{2}\sqrt{5} \right)^n \right\}$$

## 3.5 Units, Scientific Constants, and Uncertainty

In addition to manipulating exact symbolic and numeric quantities, Maple can perform computations with units and uncertainties.

Maple supports hundreds of units, for example, miles, coulombs, and bars, and provides facilities for adding custom units.

Maple has a library of hundreds of scientific constants with units, including element and isotope properties.

To support computations with uncertainties, Maple propagates errors through computations.

## Units

The **Units** package in Maple provides a library of units, and facilities for using units in computations. It is fully extensible so that you can add units as required.

**Note:** Some unit operations are available as task templates (see **Tools>Tasks>Browse**) and through context menus.

### Overview of Units

A *dimension* is a measurable quantity, for example, length or force. The set of dimensions that are fundamental and independent are known as *base dimensions*.

In Maple, the base dimensions include length, mass, time, electric current, thermodynamic temperature, amount of substance, luminous intensity, information, and currency. For a complete list, run `Units[GetDimensions]()`.

Complex dimensions measure other quantities in terms of a combination of base dimensions. For example, the complex dimension force is a measurement

of  $\frac{\text{mass length}}{\text{time}^2}$ .

Each dimension, base or complex, has associated units. (Base units measure a base dimension. Complex units measure a complex dimension.) Maple supports over forty units of length, including feet, miles, meters, angstroms, microns, and astronomical units. A length must be measured in terms of a unit, for example, a length of 2 parsecs.

Table 3.4 lists some dimensions, their corresponding base dimensions, and example units.

**Table 3.4: Sample Dimensions**

Dimension	Base Dimensions	Example Units
Time	time	second, minute, hour, day, week, month, year, millennium, blink, lune
Energy	$\frac{\text{length}^2 \text{ mass}}{\text{time}^2}$	joule, electron volt, erg, watt hour, calorie, Calorie, British thermal unit
Electric potential	$\frac{\text{length}^2 \text{ mass}}{\text{time}^3 \text{ electric current}}$	volt, abvolt, statvolt

For the complete list of units (and their contexts and symbols) available for a dimension, refer to the corresponding help page, for example, the **?Units/length** help page for the units of length.

Each unit has a *context*. The context differentiates between different definitions of the unit. For example, the standard and US survey miles are different units of length, and the second is a unit of time and of angle. You can specify the context for a unit by appending the context as an index to the unit, for example, **mile[US\_survey]**. If you do not specify a context, Maple uses the default context.

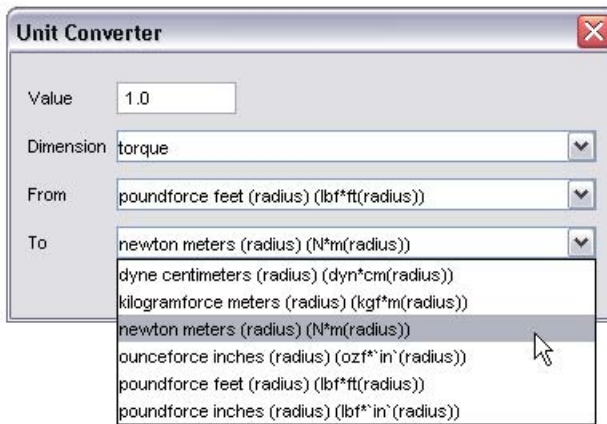
Units are collected into systems, for example, the foot-pound-second (FPS) system and international system, or *système international*, (SI). Each system has a default set of units used for measurements. In the FPS system, the foot, pound, and second are used to measure the dimensions of length, mass, and time. The unit of speed is the foot/second. In SI, the meter, kilogram, and second are used to measure the dimensions of length, mass, and time. The units of speed, magnetic flux, and power are the meter/second, weber, and watt.

## Conversions

To convert a value measured in a unit to the corresponding value in a different unit, use the **Unit Converter**.

- From the **Tools** menu, select **Assistants**, and then **Unit Converter**.

The **Unit Converter Assistant** (Figure 3.6) opens.



**Figure 3.6: Unit Converter Assistant**

#### To perform a conversion:

1. In the **Value** text field, enter the numeric value to convert.
2. In the **Dimension** drop-down list, select the dimensions of the unit.
3. From the **From** and **To** menus, select the original unit and the unit to which to convert.
4. Click **Insert**.

Maple inserts the corresponding **convert/units** command into the document.

```
> convert(1.0, 'units', 'lbf*ft(radius)', 'N m(radius)')
```

1.355817948

**Important:** Using the **Unit Converter**, you can convert temperatures and temperature changes.

- To perform a *temperature* conversion, in the **Dimension** drop-down list, select **temperature(absolute)**.

- To perform a *temperature change* conversion, in the **Dimension** dropdown list, select **temperature(relative)**.

To convert temperature changes, the **Unit Converter** uses the **convert/units** command. For example, an increase of 32 degrees Fahrenheit corresponds to an increase of almost 18 degrees Celsius.

```
> convert(32.0, 'units', 'degF', 'degC')
```

17.77777778

To convert absolute temperatures, the **Unit Converter** uses the **convert/temperature** command. For example, 32 degrees Fahrenheit corresponds to 0 degrees Celsius.

```
> convert(32, 'temperature', 'degF', 'degC')
```

0

### Applying Units to an Expression

To insert a unit, use the **Units** palettes. The **Units (FPS)** palette (Figure 3.7) contains important units from the foot-pound-second system of units. The **Units (SI)** palette (Figure 3.8) contains important units from the international system of units.

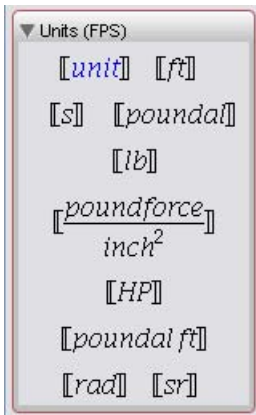


Figure 3.7: Units (FPS) Palette

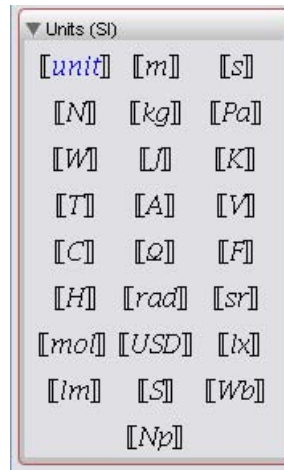


Figure 3.8: Units (SI) Palette

**To insert a unit:**

- In a **Units** palette, click a unit symbol.

> 3  $[[ft]]$

3  $[[ft]]$

**To insert a unit that is unavailable in the palettes:**

1. In a **Units** palette, click the **unit** symbol  $[[unit]]$ . Maple inserts a **Unit** object with the placeholder selected.
2. In the placeholder, enter the unit name (or symbol).

For example, to enter 0.01 standard (the default context) miles, you can specify the unit name, **mile**, or symbol, **mi**.

> 0.01  $[[mile]]$

0.01  $[[mi]]$

The context of a unit is displayed only if it is not the default context.

**Important:** In 1-D Math input, the quantity and unit (entered using the top-level **Unit** command) are a product, not a single entity. The following calling sequences define different expressions.

> **1\*Unit(m)/(2\*Unit(s));**

$\frac{1}{2} \frac{[[m]]}{[[s]]}$

> **1\*Unit(m)/2\*Unit(s);**

$\frac{1}{2} \frac{[[m]]}{[[s]]}$

Some units support prefixes. For example, SI units support prefixes to names and symbols. You can specify 1000 meters using **kilometer** or **km**. For more information, refer to the **?Units/prefixes** help page.

> 1.5  $[[km_{SI}]]$

1.5  $[[km]]$

## Performing Computations with Units

In the default Maple environment, you cannot perform computations with quantities that have units. You can perform only unit conversions. For more information about the default environment, refer to the **?Units/default** help page.

To compute with expressions that have units, you must load a **Units** environment, **Natural** or **Standard**. It is recommended that you use the **Standard** environment.

> *with(Units[Standard])* :

In the **Standard Units** environment, commands that support expressions with units return results with the correct units.

$$> \text{area} := 3[\text{ft}] \cdot \frac{1}{8}[\text{mile}]$$

$$\text{area} := \frac{14370939}{78125} [\text{m}^2]$$

$$> \frac{(-12 \sin(x) + x^2)[\text{m}]}{[\text{s}]}$$

$$(-12 \sin(x) + x^2) \left[ \frac{\text{m}}{\text{s}} \right] \quad (3.10)$$

$$> \text{int}((3.10), x[\text{s}])$$

$$\left( 12 \cos(x) + \frac{1}{3}x^3 \right) [\text{m}] \quad (3.11)$$

$$> \text{diff}((3.11), x[\text{s}])$$

$$(-12 \sin(x) + x^2) \left[ \frac{\text{m}}{\text{s}} \right]$$

For information on differentiation and integration, see *Calculus* (page 153).

### Changing the Current System of Units

If a computation includes multiple units, all units are expressed using units from the current system of units.

$$> 132.25[\text{mile}]$$

$$132.25 [\text{mi}] \quad (3.12)$$

By default, Maple uses the SI system of units, in which length is measured in meters and time is measured in seconds.



>  $\frac{(3.12)}{3}$  `[[hour]]`

19.70701333 `[[ $\frac{m}{s}$ ]]`

To view the name of the default system of units, use the **Units[UsingSystem]** command.

> `with(Units) :`

> `UsingSystem()`

*SI*

To change the system of units, use the **Units[UseSystem]** command.

> `UseSystem(FPS) :`

>  $(3.12) \cdot 3$  `[[m]]`  $\cdot 1.1$  `[[kg]]`

.1666720741  $10^8$  `[[ft2 lb]]`

## Extensibility

You can extend the set of:

- Base dimensions and units
- Complex dimensions
- Complex units
- Systems of units

For more information, refer to the **?Units[AddBaseUnit]**, **?Units[AddDimension]**, **?Units[AddUnit]**, and **?Units[AddSystem]** help pages.

For more information about units, refer to the **?Units** help page.

## Scientific Constants and Element Properties

Computations often require not only units (see *Units (page 97)*), but also the values of scientific constants, including properties of elements and their isotopes. Maple supports computations with scientific constants. You can use the built-in constants and add custom constants.

### Overview of Scientific Constants and Element Properties

The **ScientificConstants** package provides the values of constant physical quantities, for example, the velocity of light and the atomic weight of sodium. The **ScientificConstants** package also provides the units for the constant values, allowing for greater understanding of the equation as well as unit-matching for error checking of the solution.

The quantities available in the **ScientificConstants** package are divided into two distinct categories.

- Physical constants
- Chemical element (and isotope) properties

### Scientific Constants

Maple contains many built-in scientific constants, which you can easily include in your computations.

### List of Scientific Constants

You have access to scientific constants important in engineering, physics, chemistry, and other fields. Table 3.5 lists some of the supported constants. For a complete list of scientific constants, refer to the **?ScientificConstants/PhysicalConstants** help page.

**Table 3.5: Scientific Constants**

Name	Symbol
Newtonian_constant_of_gravitation	G
Planck_constant	h
elementary_charge	e
Bohr_radius	a[0]
deuteron_magnetic_moment	mu[d]
Avogadro_constant	N[A]
Faraday_constant	F

You can specify a constant using either its name or symbol.

### Accessing Constant Definition

The **GetConstant** command in the **ScientificConstants** package returns the complete definition of a constant.

To view the definition of the Newtonian gravitational constant, specify the symbol **G** (or its name) in a call to the **GetConstant** command.

> *with(ScientificConstants):*

> *GetConstant('G')*

*Newtonian\_constant\_of\_gravitation*

*,symbol = G,*

*value = .6673 10<sup>-10</sup>, uncertainty = .10 10<sup>-12</sup>, units =  $\frac{m^3}{kg s^2}$*

For information on accessing a constant's value, units, or uncertainty, see *Value, Units, and Uncertainty (page 108)*.

## Element Properties

Maple also contains element properties and isotope properties.

### Elements

Maple supports the first 112 elements of the periodic table, plus elements number 114 and 116. Each element has a unique name, atomic number, and chemical symbol. You can specify an element using any of these labels. For a complete list of supported elements, refer to the **?ScientificConstants/elements** help page.

Maple supports key element properties, including atomic weight (**atomicweight**), electron affinity (**electronaffinity**), and density. For a complete list of element properties, refer to the **?ScientificConstants/properties** help page.

### Isotopes

Isotopes, variant forms of an element that contain the same number of protons but a different number of neutrons, exist for many elements.

To see the list of supported isotopes for an element, use the **GetIsotopes** command.

```
> GetIsotopes('element' = 'Li')
```

```
Li4, Li5, Li6, Li7, Li8, Li9, Li10, Li11, Li12
```

Maple supports isotopes and has a distinct set of properties for isotopes, including abundance, binding energy (**bindingenergy**), and mass excess (**massexcess**). For a complete list of isotope properties, refer to the **?ScientificConstants/properties** help page.

### Accessing an Element or Isotope Property Definition

The **GetElement** command in the **ScientificConstants** package returns the complete definition of an element or isotope.

> *GetElement('Li')*

```
3, symbol = Li, name = lithium, names = {lithium},  
  atomicweight = [ value = 6.941, uncertainty = 0.002, units = u ]  
  , ionizationenergy  
  = [ value = 5.3917, uncertainty = undefined, units = eV ],  
  density = [ value = 0.534, uncertainty = undefined,  
  units =  $\frac{\text{g}}{\text{cm}^3}$  ],  
  electronegativity  
  = [ value = 0.98, uncertainty = undefined, units = 1 ],  
  electronaffinity = [ value = 0.6180, uncertainty = 0.0005,  
  units = eV ],  
  boilingpoint = [ value = 1615., uncertainty = undefined,  
  units = K ],  
  meltingpoint = [ value = 453.65, uncertainty = undefined,  
  units = K ]
```

> *GetElement('Li[4]')*

```
Li4, atomicmass = [ value = .4027182329 107,  
  uncertainty = 227.733, units = uu ],  
  massexcess = [ value = 25320.173, uncertainty = 212.132,  
  units = keV ],  
  bindingenergy = [ value = 4618.058, uncertainty = 212.132,  
  units = keV ]
```

### Value, Units, and Uncertainty

To use constants or element properties, you must first construct a **Scientific-Constants** object.

To construct a scientific constant, use the **Constant** command.

> *G := Constant('G')*

$$G := \text{Constant}(G)$$

To construct an element (or isotope) property, use the **Element** command.

```
> LiAtomicWeight := Element('Li', atomicweight)
```

$$LiAtomicWeight := \text{Element}(Li, atomicweight)$$

## Value

To obtain the value of a **ScientificConstants** object, use the **evalf** command.

```
> evalf(G)
```

$$.6673 \cdot 10^{-10}$$

```
> evalf(LiAtomicWeight)
```

$$.1152580953 \cdot 10^{-25}$$

**Note:** The value returned depends on the current system of units. For information on controlling the system of units, see *Changing the Current System of Units* (page 103).

## Units

To obtain the units for a **ScientificConstants** object, use the **GetUnit** command.

```
> GetUnit(G)
```

$$\left[ \frac{m^3}{kg \cdot s^2} \right]$$

```
> GetUnit(LiAtomicWeight)
```

$[[kg]]$ 

For information on changing the default system of units, for example, from SI to foot-pound-second, see *Changing the Current System of Units (page 103)*.

## Value and Units

If performing computations with units, you can access the value and units for a **ScientificConstants** object by specifying the **units** option when constructing the object, and then evaluating the object.

> `evalf(Constant('G', units))`

$$.6673 \cdot 10^{-10} \left[ \frac{m^3}{kg \cdot s^2} \right]$$

> `evalf(Element('Li[5]', atomicmass, units))`

$$.8323520514 \cdot 10^{-26} [[kg]]$$

## Uncertainty

The value of a constant is often determined by direct measurement or derived from measured values. Hence, it has an associated uncertainty. To obtain the uncertainty in the value of a **ScientificConstants** object, use the **GetError** command.

> `GetError(G)`

$$.10 \cdot 10^{-12}$$

> `GetError(LiAtomicWeight)`

$$.3321080400 \cdot 10^{-29}$$

## Performing Computations

You can use constant values in any computation. To use constant values with units, use a **Units** environment as described in *Performing Computations with Units* (page 102). For information on computing with quantities that have an uncertainty, see the following section.

## Modification and Extensibility

You can change the definition of a scientific constant or element (or isotope) property.

For more information, refer to the **?ScientificConstants[ModifyConstant]** and **?ScientificConstants[ModifyElement]** help pages.

You can extend the set of:

- Constants
- Elements (and isotopes)
- Element (or isotope) properties

For more information, refer to the **?ScientificConstants[AddConstant]**, **?ScientificConstants[AddElement]**, and **?ScientificConstants[AddProperty]** help pages.

For more information about constants, refer to the **?ScientificConstants** help page.

## Uncertainty Propagation

Some computations involve uncertainties (or errors). Using the **ScientificErrorAnalysis** package, you can propagate the uncertainty in these values through the computation to indicate the possible error in the final result.

The **ScientificErrorAnalysis** package does not perform interval arithmetic. That is, the error of an object does not represent an interval in which possible values must be contained. (To perform interval arithmetic, use the **Tolerances** package. For more information, refer to the **?Tolerances** help page.) The



quantities represent unknown values with a central tendency. For more information on central tendency, refer to any text on error analysis for the physical sciences or engineering.

### Quantities with Uncertainty

**Creating** To construct quantities with uncertainty, use the **Quantity** command. You must specify the value and uncertainty. The uncertainty can be defined absolutely, relatively, or in units of the last digit. For more information on uncertainty specification, refer to the **?ScientificErrorAnalysis[Quantity]** help page.

The output displays the value and uncertainty of the quantity.

```
> with(ScientificConstants): with(ScientificErrorAnalysis):
```

```
> Quantity(105, 1.2)
```

```
Quantity(105, 1.2)
```

```
> Quantity(105, 0.03, 'relative')
```

```
Quantity(105, 3.15) (3.13)
```

To specify the error in units of the last digit, the value must be of floating-point type.

```
> Quantity(105.0, 12, 'uld')
```

```
Quantity(105.0, 12)
```

To access the value and uncertainty of a quantity with uncertainty, use the **evalf** and **ScientificErrorAnalysis[GetError]** commands.

```
> evalf((3.13))
```

105.

```
> GetError(3.13)
```

3.15

**Rounding** To round the error of a quantity with uncertainty, use the **ApplyRule** command. For a description of the predefined rounding rules, refer to the **?ScientificErrorAnalysis/rules** help page.

```
> GetError(ApplyRule(3.13), 'round[2]')
```

3.2

**Units** Quantities with errors can have units. For example, the scientific constants and element (and isotope) properties in the **ScientificConstants** packages are quantities with errors and units.

To construct a new quantity with units and an uncertainty, include units in the **Quantity** calling sequence.

For an absolute error, you must specify the units in both the value and error.

```
> with(Units[Standard]):
```

```
> Quantity(3.5 [m], 0.1 [m])
```

```
Quantity(3.5 [m], 0.1 [m])
```

For a relative error, you can specify the units in only the value.

```
> Quantity(3.5 [m], 0.1, 'relative')
```

```
Quantity(3.5 [m], 0.35 [m])
```

For information on the correlation between, variance of, and covariance between quantities with uncertainty, refer to the **?ScientificErrorAnalysis** help page.

### Performing Computations with Quantities with Uncertainty

Many Maple commands support quantities with uncertainty.

```
> q1 := Quantity(31., 2.):
```

```
> q2 := Quantity(20., 1.):
```

Compute the value of the derivative of  $q1 \cdot x^2 + \sin(q2 \cdot x)$  at  $x = \sin(\pi/4)$ .

```
> d1 := diff(q1·x2 + sin(q2·x), x)
```

```
      d1 := 2 Quantity(31., 2.) x  
           + cos(Quantity(20., 1.) x) Quantity(20., 1.)
```

```
> d2 := eval(d1, x = sin( $\frac{\pi}{4}$ )):
```

To convert the solution to a single quantity with uncertainty, use the **combine/errors** command.

```
> result := combine(d2, 'errors');
```

The value of the result is:

```
> evalf(result)
```

43.74124725

The uncertainty of the result is:

```
> GetError(result)
```

14.42690612

### Additional Information

For information on topics including:

- Creating new rounding rules
- Setting the default rounding rule
- Creating a new interface to quantities with uncertainty

refer to the `?ScientificErrorAnalysis` help page.

## 3.6 Restricting the Domain

By default, Maple computes in the complex number system. Most computations are performed without any restrictions or assumptions on the variables. Maple often returns results that are extraneous or unsimplified when computing in the field of complex numbers. Using restrictions, you can more easily and efficiently perform computations in a smaller domain.

Maple has facilities for performing computations in the real number system and for applying assumptions to variables.

### Real Number Domain

To force Maple to perform computations in the field of real numbers, use the `RealDomain` package.

The `RealDomain` package contains a small subset of Maple commands related to basic precalculus and calculus mathematics, for example, `arccos`, `limit`, and `log`, and the symbolic manipulation of expressions and formulae, for example, `expand`, `eval`, and `solve`. For a complete list of commands, refer to the `?RealDomain` help page.

After you load the **RealDomain** package, Maple assumes that all variables are real. Commands return simplified results appropriate to the field of real numbers.

> *with(RealDomain):*

> *simplify*( $\sqrt{x^2}$ )

$|x|$

>  $\ln(e^x)$

$x$

Some commands that generally return **NULL** instead return a numeric result when you use the **RealDomain** package.

>  $(-32)^{\left(\frac{1}{5}\right)}$

$-2$

Complex return values are excluded or replaced by **undefined**.

> *solve*( $x^2 = -1$ )

> *arcsin*( $e^2$ )

*undefined*

## Assumptions on Variables

To simplify problem solving, it is recommended that you always apply any known assumptions to variables. You can impose assumptions using the **assume** command. To apply assumptions for a single computation, use the **assuming** command.

**Note:** The **assume** and **assuming** commands are not supported by the **RealDomain** package.

### The **assume** Command

You can use the **assume** command to set variable properties, for example, **x::real**, and relationships between variables, for example, **x < 0** or **x < y**. For information on valid properties, refer to the **?assume** help page. For information on the double colon (**::**) operator, refer to the **?type** help page.

The **assume** command allows improved simplification of symbolic expressions, especially multiple-valued functions, for example, computing the square root.

To assume that **x** is a positive real number, use the following calling sequence.

Then compute the square root of  $x^2$ .

```
> assume(0 < x):  $\sqrt{x^2}$ 
                    x~
```

The trailing tilde (~) on the name **x** indicates that it carries assumptions.

When you use the **assume** command to place another assumption on **x**, all previous assumptions are removed.

```
> assume(x < 0):  $\sqrt{x^2}$ 
```

$-x\sim$

**Displaying Assumptions** To view the assumptions on an expression, use the **about** command.

```
> about(x)
```

*Originally  $x$ , renamed  $x\sim$ : is assumed to be:  $\text{RealRange}(-\text{infinity}, \text{Open}(0))$*

**Imposing Multiple Assumptions** To simultaneously impose multiple conditions on an expression, specify multiple arguments in the **assume** calling sequence.

```
> assume(0 < x, x < 2)
```

To specify additional assumptions without replacing previous assumptions, use the **additionally** command. The syntax of the **additionally** calling sequence is the same as that of the **assume** command.

```
> additionally(x::integer): about(x)
```

*Originally  $x$ , renamed  $x\sim$ : is assumed to be: 1*

The only integer in the open interval **(0, 2)** is **1**.

**Testing Properties** To test whether an expression always satisfies a condition, use the **is** command.

```
> assume(15 < x, 7 < y): is(100 < x y)
```

*true*

The following test returns **false** because there are values of **x** and **y** (**x = 0**, **y = 10**) that satisfy the assumptions, but do not satisfy the relation in the **is** calling sequence.

```
> assume(x::nonnegint, 10 ≤ y): is(10 < x + y)
```

*false*

To test whether an expression can satisfy a condition, use the **coulditbe** command.

```
> coulditbe(10 < x + y)
```

*true*

**Removing Assumptions** To remove all assumptions on a variable, unassign its name.

```
> unassign('x', 'y')
```

For more information, see *Unassigning Names* (page 57).

For more information on the **assume** command, refer to the **?assume** help page.

## The assuming Command

To perform a single evaluation under assumptions on the names in an expression, use the **assuming** command.

The syntax of the assuming command is **expression assuming** *<property or relation>*. Properties and relations are introduced in *The assume Command* (page 117).

The **frac** command returns the fractional part of an expression.

```
> frac(x) assuming x :: integer
```

0

Using the **assuming** command is equivalent to imposing assumptions with the **assume** command, evaluating the expression, and then removing the assumptions.



> *about(x)*

*x: nothing known about this object*

If you do not specify the names to which to apply a property, it is applied to all names.

>  $\sqrt{\left(\frac{a}{b}\right)^2}$  *assuming positive*

$\frac{a}{b}$

Assumptions placed on names using the **assume** command are ignored by the **assuming** command, unless you include the **additionally** option.

> *assume(x < 1)*

> *is(1 - x<sup>2</sup> > 0) assuming x > 1*

*false*

> *is(1 - x<sup>2</sup> > 0) assuming additionally, x > -1*

*true*

The **assuming** command does not affect variables inside procedures. (For information on procedures, see *Procedures (page 338)*.) You must use the **assume** command.

> **f := proc(x) sqrt(a^2) + x end proc;**

**f := proc(x) sqrt(a^2) + x end proc**

> *f(1) assuming a > 0*

$$\sqrt{a^2 + 1}$$

> *assume*( $a > 0$ ):  $f(1)$

$$a + 1$$

For more information on the **assuming** command, refer to the **?assuming** help page.



# 4 Mathematical Computations

As discussed in previous chapters, Maple contains numerous built-in resources for computations.

These resources—and others on the Maplesoft Web site—are available for the areas discussed in this chapter, and many more. Your first step in solving a problem should be to review the related Maple resources available. This will help you to quickly and easily solve problems. See Table 4.1.

**Table 4.1: Maple Resources for Mathematical Computation**

Resource	Description
Point-and-click assistants	Graphical interfaces with buttons and sliders to easily perform a computation, create a plot, or perform other operations. <ul style="list-style-type: none"><li>• From the <b>Tools</b> menu, select <b>Assistants</b>.</li></ul>
Context menus	Pop-up menu of common operations for the selected object, based on its type. <ul style="list-style-type: none"><li>• Select the expression in 2-D Math input or output, and then right-click (for Macintosh, <b>Control-click</b>).</li></ul>
Palettes	Collections of related items that you can insert by clicking or dragging. Some palettes contain mathematical operations with placeholders for parameters. <ul style="list-style-type: none"><li>• From the <b>View</b> menu, select <b>Palettes</b>, and then <b>Expand Docks</b>.</li></ul>
Task templates	Set of commands with placeholders that you can use to quickly perform a task. Some tasks contain graphical components such as buttons. <ul style="list-style-type: none"><li>• From the <b>Tools</b> menu, select <b>Tasks</b>, and then <b>Browse</b>.</li></ul>
<b>FunctionAdvisor</b> command	Provides detailed information about mathematical functions, for example, definitions, identities, and mathematical properties. <ul style="list-style-type: none"><li>• Refer to the <b>?FunctionAdvisor</b> help page.</li></ul>

Resource	Description
Maple Help System	Over 5000 help pages and example worksheets with an integrated search engine. <ul style="list-style-type: none"> <li>• From the <b>Help</b> menu, select <b>Maple Help</b>.</li> </ul>
Package index help page	A complete list of the over 100 Maple packages, which contain thousands of commands. <ul style="list-style-type: none"> <li>• From the <b>Help</b> menu, select <b>Manuals, Dictionary, and more</b>, and then <b>List of Packages</b>.</li> </ul>
Command index help page	A complete list of the over 600 top-level Maple commands. <ul style="list-style-type: none"> <li>• From the <b>Help</b> menu, select <b>Manuals, Dictionary, and more</b>, and then <b>List of Commands</b>.</li> </ul>
Maplesoft Web site ( <a href="http://www.maplesoft.com">http://www.maplesoft.com</a> )	Maple Application Center - Free documents and point-and-click Maplet applications for mathematics, engineering, finance, and science. <ul style="list-style-type: none"> <li>• Visit <a href="http://www.maplesoft.com/applications">http://www.maplesoft.com/applications</a></li> </ul> Toolboxes - Add-on products from Maplesoft, for example, the Global Optimization Toolbox. <ul style="list-style-type: none"> <li>• Visit <a href="http://www.maplesoft.com/products/toolboxes">http://www.maplesoft.com/products/toolboxes</a></li> </ul> Third-Party Products - Add-on products developed by the Maple user community for specialized computation. <ul style="list-style-type: none"> <li>• Visit <a href="http://www.maplesoft.com/products/thirdparty">http://www.maplesoft.com/products/thirdparty</a></li> </ul>

For instructor and student resources, see *Table 4.10 (page 180)*.

For information on basic computations, including integer operations and solving equations, see *Performing Computations (page 65)*.

## 4.1 In This Chapter

Section	Topics
Algebra - Performing algebra computations	<ul style="list-style-type: none"> <li>• Polynomial Algebra</li> </ul>
Linear Algebra - Performing linear algebra computations	<ul style="list-style-type: none"> <li>• Creating Matrices and Vectors</li> <li>• Accessing Entries in Matrices and Vectors</li> <li>• Linear Algebra Computations</li> <li>• <b>Student LinearAlgebra</b> Package</li> </ul>
Calculus - Performing calculus computations	<ul style="list-style-type: none"> <li>• Limits</li> <li>• Differentiation</li> <li>• Series</li> <li>• Integration</li> <li>• Differential Equations</li> <li>• Calculus Packages</li> </ul>
Optimization - Performing optimization computations using the <b>Optimization</b> package	<ul style="list-style-type: none"> <li>• Point-and-Click Interface</li> <li>• Efficient Computation</li> <li>• MPS(X) File Support</li> </ul>
Statistics - Performing statistics computations using the <b>Statistics</b> package	<ul style="list-style-type: none"> <li>• Probability Distributions and Random Variables</li> <li>• Statistical Computations</li> <li>• Plotting</li> </ul>
Teaching and Learning with Maple - Student and Instructor resources for using Maple in an academic setting	<ul style="list-style-type: none"> <li>• Table of Student and Instructor Resources</li> <li>• <b>Student</b> Packages and Tutors</li> </ul>

## 4.2 Algebra

Maple contains a variety of commands that perform integer operations, such as factoring and modular arithmetic, as described in *Integer Operations* (page 71). In addition, it supports polynomial algebra.

For information on matrix and vector algebra, see *Linear Algebra* (page 135).

### Polynomial Algebra

A Maple polynomial is an expression in powers of an unknown. *Univariate* polynomials are polynomials in one unknown, for example,  $x^3 - 2x + 13$ .

*Multivariate* polynomials are polynomials in multiple unknowns, such as

$$x^3y - \frac{3}{2}xy^2 + 7x.$$

The coefficients can be integers, rational numbers, irrational numbers, floating-point numbers, complex numbers, variables, or a combination of these types.

$$> ax^2 + 7x - \frac{b}{2}$$

$$ax^2 + 7x - \frac{1}{2}b$$

### Arithmetic

The polynomial arithmetic operators are the standard Maple arithmetic operators excluding the division operator (*/*). (The division operator accepts polynomial arguments, but does not perform *polynomial division*.)

Polynomial division is an important operation. The **quo** and **rem** commands find the quotient and remainder of a polynomial division. See Table 4.2. (The **iquo** and **irem** commands find the quotient and remainder of an integer division. For more information, see *Integer Operations* (page 71).)

**Table 4.2: Polynomial Arithmetic Operators**

Operation	Operator	Example
Addition	+	$> (x^2 + 1) + (3x^3 - 5x + 2)$ $x^2 + 3 + 3x^3 - 5x$
Subtraction	-	$> (x^2 + 1) - (3x^3 - 5x + 2)$ $x^2 - 1 - 3x^3 + 5x$
Multiplication <sup>1</sup>	*	$> (x^2 + 1) \cdot (3x^3 - 5x + 2)$ $(x^2 + 1)(3x^3 - 5x + 2)$
Division: Quotient and Remainder	<b>quo</b> <b>rem</b>	$> \text{quo}(2x^2 + x - 3, 3x + 5, x)$ $\frac{2}{3}x - \frac{7}{9}$ $> \text{rem}(2x^2 + x - 3, 3x + 5, x)$ $\frac{8}{9}$
Exponentiation <sup>2</sup>	^	$> (x^2 + 1)^3$ $(x^2 + 1)^3$

<sup>1</sup>You can specify multiplication explicitly by entering \*, which displays in 2-D Math as  $\cdot$ . In 2-D Math, you can also implicitly multiply by placing a space character between two expressions. In some cases, the space character is optional. For example, Maple interprets a number followed by a name as an implicit multiplication.

<sup>2</sup>In 2-D Math, exponents display as superscripts.



To expand a polynomial, use the **expand** command.

```
> expand(3x^2*(3x+5)-(x^2-2))  
  
          9x^3 + 14x^2 + 2
```

If you need to determine whether one polynomial divides another, but do not need the quotient, use the **divide** command. The **divide** command tests for exact polynomial division.

```
> divide(x^4y^2+x^3y^2-x^2y^2+13x^2+13x-13+y*x^2+x*y-y, x^2  
      +x-1)  
  
          true
```

**Important:** You must insert a space character or a multiplication operator (  $\cdot$  ) between adjacent variables names. Otherwise, they are interpreted as a single variable.

For example,  $x$  does not divide the single variable  $xy$ .

```
> divide(xy,x)  
  
          false
```

But,  $x$  divides the product of  $x$  and  $y$ .

```
> divide(xy,x); divide(x*y,x)  
  
          true  
  
          true
```

For information on polynomial arithmetic over finite rings and fields, refer to the **?mod** help page.

## Sorting Terms

To sort the terms of a polynomial, use the **sort** command.

>  $p1 := x^2 + x^3 - x + x^4$

$$p1 := x^2 + x^3 - x + x^4$$

>  $sort(p1)$

$$x^4 + x^3 + x^2 - x$$

**Note:** The **sort** command returns the sorted polynomial, and updates the order of the terms in the polynomial.

The terms of **p1** are sorted.

>  $p1$

$$x^4 + x^3 + x^2 - x$$

To specify the unknowns of the polynomial and their ordering, include a list of names.

>  $sort(a^2x^3 + x^2 + x \cdot a + a + b, [a])$

$$x^3 a^2 + x a + a + x^2 + b$$

>  $sort(a^2x^3 + x^2 + x \cdot a + a + b, [x, b])$

$$a^2 x^3 + x^2 + a x + b + a$$

By default, the **sort** command sorts a polynomial by decreasing *total degree* of the terms.

>  $p2:=x^3+y^3+x^2y^2$ :

>  $sort(p2, [x,y])$

$$x^2y^2 + x^3 + y^3$$

The first term has total degree 4. The other two terms have total degree 3. The order of the final two terms is determined by the order of their names in the list.

To sort the terms by *pure lexicographic order*, that is, first by decreasing order of the first unknown in the list option, and then by decreasing order of the next unknown in the list option, specify the '**plex**' option.

>  $sort(p2, [x,y], 'plex')$

$$x^3 + x^2y^2 + y^3$$

For information on enclosing keywords in right single quotes ('), see *Delaying Evaluation* (page 317).

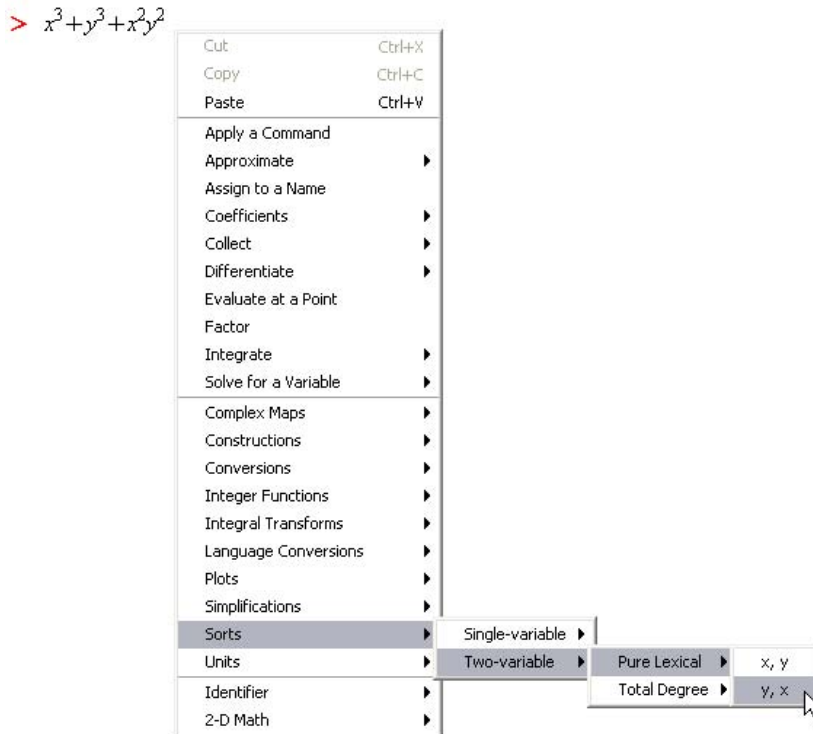
The first term has a power of  $x$  to the 3. The second, a power of  $x$  to the 2. The third, a power of  $x$  to the 0.

Using context menus, you can perform operations, such as sorting, for polynomials and many other Maple objects.

### To sort a polynomial:

1. Right-click (**Control**-click, for Macintosh) the polynomial.
2. The context menu displays. From the **Sorts** menu, select:
  - **Single-variable**, and then the unknown
  - **Two-variable** (or **Three-variable**), **Pure Lexical** or **Total Degree**, and then the sort priority of the unknowns

See Figure 4.1.



**Figure 4.1: Sorting a Polynomial Using a Context Menu**

Maple sorts the polynomial.

In Worksheet mode, Maple inserts the calling sequence that performs the sort followed by the sorted polynomial.

>  $x^3 + y^3 + x^2y^2$

> `sort( x^3+y^3+x^2*y^2, [y, x], 'plex' )`

$$y^3 + y^2x^2 + x^3$$

You can use context menus to perform operations on 2-D Math content including output. For more information, see *Context Menus (page 20)* (for Document mode) or *Context Menus (page 46)* (for Worksheet mode).

## Collecting Terms

To collect the terms of polynomial, use the **collect** command.

$$\begin{aligned} > \text{collect}\left(2axy + cx^2y - zy^2 + az - 13by + \frac{3y^2}{x}, y\right) \\ & \left(-z + 3\frac{1}{x}\right)y^2 + (2ax + x^2c - 13b)y + az \end{aligned}$$

## Coefficients and Degrees

Maple has several commands that return coefficient and degree values for a polynomial. See Table 4.3.

**Table 4.3: Polynomial Coefficient and Degree Commands**

Command	Description	Example
<b>coeff</b>	Coefficient of specified degree term	$\begin{aligned} > \text{coeff}\left(\frac{1}{2}x^3 - 2x + 5, x^3\right) \\ & \frac{1}{2} \end{aligned}$
<b>lcoeff</b>	Leading coefficient	$\begin{aligned} > \text{lcoeff}\left(\frac{1}{2}x^3 - 2x + 5\right) \\ & \frac{1}{2} \end{aligned}$

Command	Description	Example
<b>tcoeff</b>	Trailing coefficient	$\text{tcoeff}\left(\frac{1}{2}x^3 - 2x + 5\right)$ 5
<b>coeffs</b>	Sequence of all coefficients in increasing degree order. <b>Note:</b> It does not return zero coefficients.	$\text{coeffs}\left(\frac{1}{2}x^3 - 2x + 5\right)$ 5, -2, $\frac{1}{2}$
<b>degree</b>	(Highest) degree	$\text{degree}\left(\frac{1}{2}x^3 - 2x + 5\right)$ 3
<b>ldegree</b>	Lowest degree term with a non-zero coefficient	$\text{ldegree}\left(\frac{1}{2}x^3 - 2x\right)$ 1

## Factorization

To express a polynomial in fully factored form, use the **factor** command.

$\text{factor}(x^4 - 1)$

$$(x - 1)(x + 1)(x^2 + 1)$$

The **factor** command factors the polynomial over the ring implied by the coefficients, for example, integers. You can specify an algebraic number field over which to factor the polynomial. For more information, refer to the **?factor** help page. (The **ifactor** command factors an integer. For more information, see *Integer Operations* (page 71).)

To solve for the roots of a polynomial, use the **solve** command. For information on the **solve** command, see *Solving Equations and Inequalities* (page 78). (The **isolve** command solves an equation for integer solutions. For more information, see *Integer Equations* (page 94).)

## Other Commands

Table 4.4 lists other commands available for polynomial operations.

**Table 4.4: Select Other Polynomial Commands**

Command	Description
<b>content</b>	Content (multivariate polynomial)
<b>compoly</b>	Decomposition
<b>discrim</b>	Discriminant
<b>gcd</b>	Greatest common divisor (of two polynomials)
<b>gcdex</b>	Extended Euclidean algorithm (for two polynomials)
<b>CurveFitting[PolynomialInterpolation]</b> See also the <b>CurveFitting Assistant</b> (Tools>Assistants>Curve Fitting)	Interpolating polynomial (for list of points)
<b>lcm</b>	Least common multiple (of two polynomials)
<b>norm</b>	Norm
<b>prem</b>	Pseudo-remainder (of two multivariate polynomials)
<b>primpart</b>	Primitive part (multivariate polynomial)
<b>randpoly</b>	Random polynomial
<b>PolynomialTools[IsSelfReciprocal]</b>	Determine whether self-reciprocal
<b>resultant</b>	Resultant (of two polynomials)
<b>roots</b>	Exact roots (over algebraic number field)

Command	Description
<b>sqrfree</b>	Square free factorization (multivariate polynomial)

### Additional Information

**Table 4.5: Additional Polynomial Help**

Topic	Resource
General polynomial information	? <b>polynom</b> help page
<b>PolynomialTools</b> package	? <b>PolynomialTools</b> package overview help page
Algebraic manipulation of numeric polynomials	? <b>SNAP</b> (Symbolic-Numeric Algorithms for Polynomials) package overview help page
Efficient arithmetic for sparse polynomials	? <b>SDMPolynomial</b> (Sparse Distributed Multivariate Polynomial data structure) help page
Polynomial information and commands	Maple Help System Table of Contents: <b>Mathematics&gt;Algebra&gt;Polynomials</b> section

## 4.3 Linear Algebra

Linear algebra operations act on **Matrix** and **Vector** data structures.

You can perform many linear algebra operations using task templates. In the **Task Browser** (**Tools>Tasks>Browse**), expand the **Linear Algebra** folder.

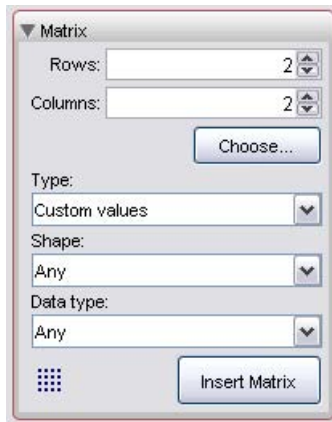
### Creating Matrices and Vectors

You can easily define matrices using the **Matrix** palette. To define vectors, use the angle-bracket (<>) notation.

#### Creating Matrices

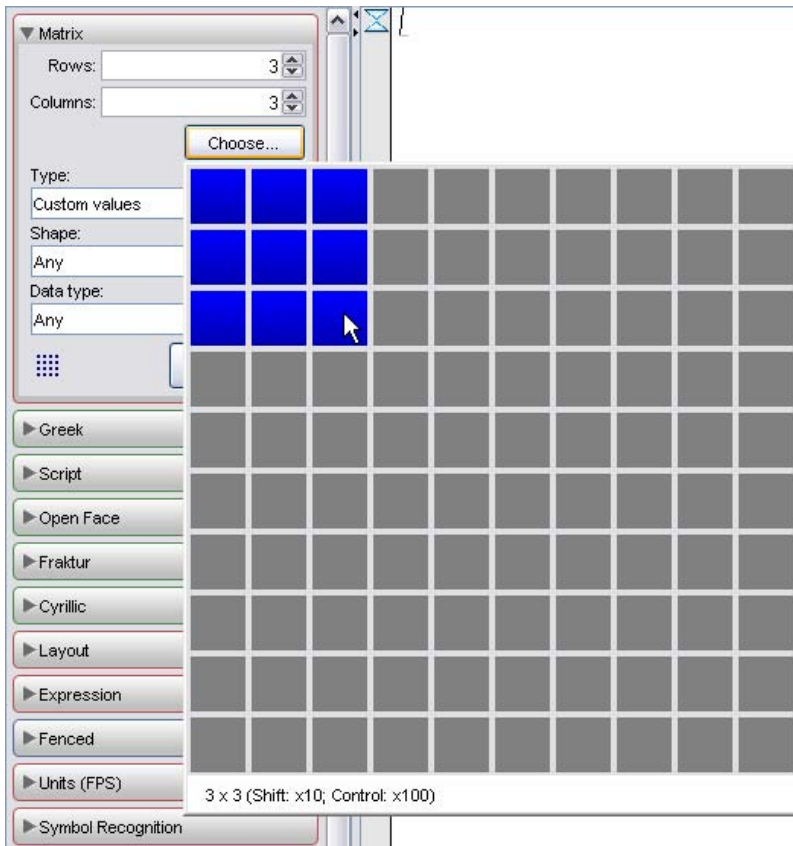
To create a matrix, use the **Matrix** palette. See Figure 4.2.





**Figure 4.2: Matrix Palette**

In the **Matrix** palette, you can specify the matrix size (see Figure 4.3) and properties. To insert a matrix, click the **Insert Matrix** button.



**Figure 4.3: Matrix Palette: Choosing the Size**

**After inserting the matrix:**

1. Enter the values of the entries. To move to the next entry placeholder, press **Tab**.
2. After specifying all entries, press **Enter**.

$$> \begin{bmatrix} 1 & e^2 & 0 \\ \pi & \sin(t) & 0 \\ 0 & \frac{87}{2} & 5e \end{bmatrix} :$$

### Creating Vectors

To create a vector, use angle brackets (<>).

To create a column vector, specify a comma-delimited sequence, <**a**, **b**, **c**>. The number of elements is inferred from the number of expressions.

> <1,2,3>

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

To create a row vector, specify a vertical-bar-delimited (|) sequence, <**a** | **b** | **c**>. The number of elements is inferred from the number of expressions.

> <1|2|3>

$$[1 \ 2 \ 3]$$

### Editing and Viewing Large Matrices and Vectors

Matrices  $10 \times 10$  and smaller, and vectors with 10 or fewer elements display in the document. Larger objects are displayed as a placeholder.

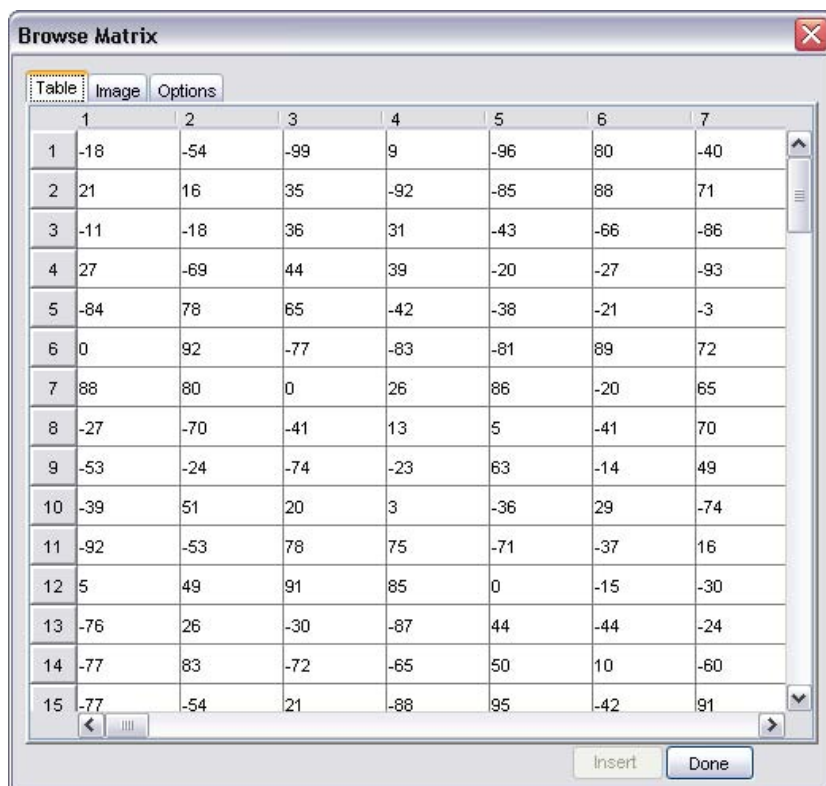
For example, insert a  $15 \times 15$  matrix.

**In the Matrix palette:**

1. Specify the dimensions: 15 rows and 15 columns.
2. In the **Type** drop-down list, select a matrix type, for example, **Custom values**.
3. Click **Insert Matrix**. Maple inserts a placeholder.

>  $\left[ \begin{array}{l} 15 \times 15 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{array} \right]:$

To edit or view a large matrix or vector, double-click the placeholder. This launches the Matrix Browser. See Figure 4.4.



**Figure 4.4: Matrix Browser**

**To specify the value of entries using the Matrix Browser:**

1. Select the **Table** tab.
2. Double-click an entry, and then edit its value. Press **Enter**.
3. Repeat for each entry to edit.
4. When you have finished updating entries, click **Done**.

You can view the matrix or vector as a table or as an image, which can be inserted into the document. For more information, refer to the **?MatrixBrowser** help page.

**To set the maximum dimension of matrices and vectors displayed inline:**

- Use the **interface** command with the **rtablesize** option.

For example, **interface(rtablesize = 15)**.

For more information, refer to the **?interface** help page.

**Creating Matrices and Vectors for Large Problems**

By default, matrices and vectors can store any values. To increase the efficiency of linear algebra computations, create matrices and vectors with properties. You must specify the properties, for example, the matrix or vector type or the data type, when defining the object.

The **Matrix** palette (Figure 4.2) supports several properties.

**To specify the matrix type:**

- Use the **Shape** and **Type** drop-down lists.

**To specify the data type:**

- Use the **Data type** drop-down list.

For example, define a diagonal matrix with small integer coefficients.

**In the Matrix palette:**

1. Specify the size of the matrix, for example,  $3 \times 3$ .
2. In the **Shapes** drop-down list, select **Diagonal**.
3. In the **Data type** drop-down list, select **integer[1]**.
4. Click the **Insert Matrix** button.
5. Enter the values in the diagonal entries.

$$> \begin{bmatrix} -23 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 32 \end{bmatrix};$$

**Note:** To create a matrix with randomly-generated entries, select the **Random Type**.

You cannot specify properties when defining vectors using the angle-bracket notation. You must use the **Vector** constructor.

**To define a column vector using the Vector constructor, specify:**

- The number of elements. If you explicitly specify all element values, this argument is not required.
- A list of expressions that define the element values.
- Parameters such as **shape**, **datatype**, and **fill** that set properties of the vector.

The following two calling sequences are equivalent.

> *Vector*([0, 0, 0])

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

> *Vector*(3, 'shape'='zero')

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

To create a row vector using the **Vector** constructor, include **row** as an index.

> `Vector[row](3, 'fill' = 1)`

`[1 1 1]`

> `Vector[row]([127, 0, 34], 'datatype' = 'integer[1])`

`[127 0 34]`

The **Matrix** palette does not support some properties. To set all properties, use the **Matrix** constructor.

**To define a matrix using the Matrix constructor, specify:**

- The number of rows and columns. If you explicitly specify all element values, these arguments are not required.
- A list of lists that define the element values row-wise.
- Parameters such as **shape**, **datatype**, and **fill** that set properties of the matrix.

For example:

> `Matrix([[1, 2, 3], [4, 5, 6]])`

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

The **Matrix** palette cannot fill the matrix with an arbitrary value. Use the **fill** parameter.

> `Matrix(3, 4, [[1, 2, 3], [4, 5, 6]], 'fill' = 2 + i)`



$$\begin{bmatrix} 1 & 2 & 3 & 2+i \\ 4 & 5 & 6 & 2+i \\ 2+i & 2+i & 2+i & 2+i \end{bmatrix}$$

For more information on the constructors, including other calling sequence syntaxes and parameters, refer to the **?storage**, **?Matrix**, and **?Vector** help pages.

See also *Numeric Computations* (page 152).

## Accessing Entries in Matrices and Vectors

To select an entry in a vector, enter the vector name with a non-zero integer index.

```
> a := <85.3, 47.1, 59.9, 38.1>
```

$$a := \begin{bmatrix} 85.3 \\ 47.1 \\ 59.9 \\ 38.1 \end{bmatrix}$$

```
> a[1]
```

85.3

Negative integers select entries from the end of the vector.

```
> a[-1]
```

38.1

To create a Vector consisting of multiple entries, specify a list or range of integers in the index. For more information, refer to the **?list** and **?range** help pages.

>  $a[[1, 2]]$

$$\begin{bmatrix} 85.3 \\ 47.1 \end{bmatrix}$$

>  $a[2..4]$

$$\begin{bmatrix} 47.1 \\ 59.9 \\ 38.1 \end{bmatrix}$$

Similarly, you can access submatrices using an index. In the following two-dimensional matrix, the first entry selects rows and the second, columns.

$$> b := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(t) & -\sin(t) \\ 0 & \sin(t) & \cos(t) \end{bmatrix} :$$

>  $b[2..-1, 2..-1]$

$$\begin{bmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{bmatrix}$$

## Linear Algebra Computations

You can perform matrix and vector computations using context menus and the **LinearAlgebra** package.

## Matrix Arithmetic

The matrix and vector arithmetic operators are the standard Maple arithmetic operators up to the following two differences.

- The scalar multiplication operator is the asterisk (\*), which displays in math as  $\cdot$ . The noncommutative matrix and vector multiplication operator is the period (.).
- There is no division operator (/) for matrix algebra. (You can construct the inverse of a matrix using the exponent  $-1$ .)

See Table 4.6.

$$> a := \begin{bmatrix} 93 & 43 \\ 19 & 37 \end{bmatrix}; b := \begin{bmatrix} 48 & 20 \\ 19 & 37 \end{bmatrix}; c := \langle 23, 6 \rangle;$$

**Table 4.6: Matrix and Vector Arithmetic Operators**

Operation	Operator	Example
Addition	+	$> a+b$ $\begin{bmatrix} 141 & 63 \\ 38 & 74 \end{bmatrix}$
Subtraction	-	$> a-b$ $\begin{bmatrix} 45 & 23 \\ 0 & 0 \end{bmatrix}$

Operation	Operator	Example
Multiplication	·	<p>&gt; <math>ac</math></p> $\begin{bmatrix} 2397 \\ 659 \end{bmatrix}$
Scalar Multiplication <sup>1</sup>	*	<p>&gt; <math>12a</math></p> $\begin{bmatrix} 1116 & 516 \\ 228 & 444 \end{bmatrix}$ <p>&gt; <math>c \cdot 4</math></p> $\begin{bmatrix} 92 \\ 24 \end{bmatrix}$
Exponentiation <sup>2</sup>	^	<p>&gt; <math>a^3</math></p> $\begin{bmatrix} 986548 & 613868 \\ 271244 & 187092 \end{bmatrix}$ <p>&gt; <math>b^{-1}</math></p> $\begin{bmatrix} \frac{37}{1396} & \frac{-5}{349} \\ \frac{-19}{1396} & \frac{12}{349} \end{bmatrix}$
<p><sup>1</sup> You can specify scalar multiplication explicitly by entering *, which displays in 2-D Math as ·. In 2-D Math, you can also implicitly multiply a scalar and a matrix or vector by placing a space character between them. In some cases, the space character is optional. For example, Maple interprets a number followed by a name as an implicit multiplication.</p> <p><sup>2</sup> In 2-D Math, exponents display as superscripts.</p>		

A few additional matrix and vector operators are listed in Table 4.7.

Define two column vectors.

```
> d := <1, 2, 3> : e := <4, 5, 6> :
```

**Table 4.7: Select Matrix and Vector Operators**

Operation	Operator	Example
Transpose	$\wedge\%T^1$	<pre>&gt; d<sup>%T</sup></pre> $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
Hermitian Transpose	$\wedge\%H^1$	<pre>&gt; <math>\begin{bmatrix} i &amp; -2i \\ 3 + 4i &amp; 2 - i \end{bmatrix}</math><sup>%H</sup></pre> $\begin{bmatrix} -i & 3 - 4i \\ 2i & 2 + i \end{bmatrix}$
Cross Product (3-D vectors only)	$\&x^2$	<pre>&gt; with(LinearAlgebra):</pre> <pre>&gt; d &amp;x e</pre> $\begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$

<sup>1</sup>Exponential operators display in 2-D Math as superscripts.

<sup>2</sup>After loading the **LinearAlgebra** package, the cross product operator is available as the infix operator **&x**. Otherwise, it is available as the **LinearAlgebra[CrossProduct]** command.

For information on matrix arithmetic over finite rings and fields, refer to the **?mod** help page.

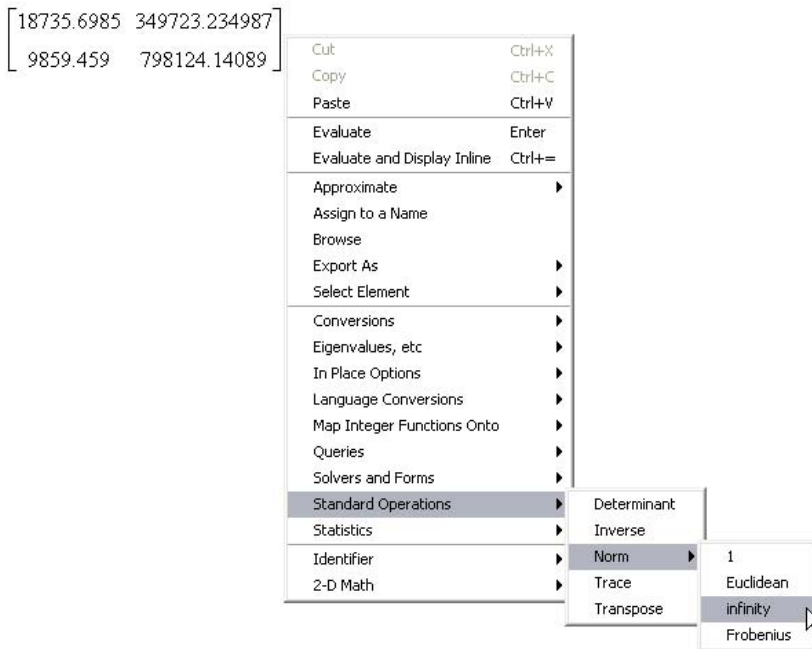
## Point-and-Click Interaction

Using context menus, you can perform many matrix and vector operations.

Matrix operations available in the context menu include the following.

- Standard operations: determinant, inverse, norm (1, Euclidean, infinity, or Frobenius), transpose, and trace
- Compute eigenvalues, eigenvectors, and singular values
- Compute the dimension or rank
- Convert to the Jordan form, or other forms
- Perform Cholesky decomposition and other decompositions

For example, compute the infinity norm of a matrix. See Figure 4.5.



**Figure 4.5: Computing the Infinity Norm of a Matrix**

In Document mode, Maple inserts a right arrow followed by the norm. See Figure 4.6.

$$\begin{bmatrix} 18735.6958 & 348723.234987 \\ 9859.459 & 798124.14089 \end{bmatrix} \rightarrow 8.079835999 \cdot 10^5$$

**Figure 4.6: Computing Norm in Document Mode**

Vector operations available in the context menu include the following.

- Compute the dimension
- Compute the norm (1, Euclidean, and infinity)
- Compute the transpose
- Select an element

For more information on context menus, see *Context Menus (page 20)* (for Document mode) or *Context Menus (page 46)* (for Worksheet mode).

## LinearAlgebra Package Commands

The **LinearAlgebra** package contains commands that construct and manipulate matrices and vectors, compute standard operations, perform queries, and solve linear algebra problems.

Table 4.8 lists some **LinearAlgebra** package commands. For a complete list, refer to the **?LinearAlgebra/Details** help page.

**Table 4.8: Select LinearAlgebra Package Commands**

Command	Description
<b>Basis</b>	Return a basis for a vector space
<b>CrossProduct</b>	Compute the cross product of two vectors
<b>DeleteRow</b>	Delete the rows of a matrix
<b>Dimension</b>	Determine the dimension of a matrix or a vector
<b>Eigenvectors</b>	Compute the eigenvalues and eigenvectors of a matrix
<b>FrobeniusForm</b>	Reduce a matrix to Frobenius form

Command	Description
<b>GaussianElimination</b>	Perform Gaussian elimination on a matrix
<b>HessenbergForm</b>	Reduce a square matrix to Hessenberg form
<b>HilbertMatrix</b>	Construct a generalized Hilbert matrix
<b>IsOrthogonal</b>	Test if a matrix is orthogonal
<b>LeastSquares</b>	Compute the least-squares approximation to $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$
<b>LinearSolve</b>	Solve the linear system $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$
<b>MatrixInverse</b>	Compute the inverse of a square matrix or pseudo-inverse of a non-square matrix
<b>QRDecomposition</b>	Compute a QR factorization of a matrix
<b>RandomMatrix</b>	Construct a random matrix
<b>SylvesterMatrix</b>	Construct the Sylvester matrix of two polynomials

For information on arithmetic operations, see *Matrix Arithmetic (page 146)*.

For information on selecting entries, subvectors, and submatrices, see *Accessing Entries in Matrices and Vectors (page 144)*.

**Example** Determine a basis for the space spanned by the set of vectors  $\{(2, 13, -15), (7, -2, 13), (5, -4, 9)\}$ . Express the vector  $(25, -4, 9)$  with respect to this basis.

> *with(LinearAlgebra):*

>  $v1 := \langle 2, 13, -15 \rangle; v2 := \langle 7, -2, 13 \rangle; v3 := \langle 5, -4, 9 \rangle;$

Find a basis for the vector space spanned by these vectors, and then construct a matrix from the basis vectors.

>  $basis := Matrix(Basis([v1, v2, v3]));$



$$\mathit{basis} := \begin{bmatrix} 2 & 7 & 5 \\ 13 & -2 & -4 \\ -15 & 13 & 9 \end{bmatrix}$$

To express  $(25, -4, 9)$  in this basis, use the **LinearSolve** command.

> `LinearSolve(basis, <25, -4, 9>)`

$$\begin{bmatrix} \frac{170}{91} \\ \frac{-285}{91} \\ \frac{786}{91} \end{bmatrix}$$

## Numeric Computations

You can very efficiently perform computations on large matrices and vectors that contain floating-point data using the built-in library of numeric linear algebra routines. Some of these routines are provided by the Numerical Algorithms Group (NAG®). Maple also contains portions of the CLAPACK and optimized ATLAS libraries.

For information on performing efficient numeric computations using the **LinearAlgebra** package, refer to the **?EfficientLinearAlgebra** help page.

See also *Creating Matrices and Vectors for Large Problems* (page 141).

## Student LinearAlgebra Package

The **Student** package contains subpackages that help instructors teach concepts and allow students to visualize and explore ideas. These subpackages also contain computational commands.

In the **Student[LinearAlgebra]** subpackage, the environment differs from that of the **LinearAlgebra** package in that floating-point computations are generally performed using software precision, instead of hardware precision, and symbols are generally assumed to represent real, rather than complex, quantities. These defaults, and others, can be controlled using the **SetDefault** command. For more information, refer to the **?Student[LinearAlgebra][SetDefault]** help page.

For information on using Maple as a teaching and learning tool, see *Teaching and Learning with Maple* (page 180).

## 4.4 Calculus

The **Task Browser** (**Tools>Tasks>Browse**) contains numerous calculus task templates. For a list of tasks, navigate to one of the related folders, such as **Calculus**, **Differential Equations**, **Multivariate Calculus**, or **Vector Calculus**.

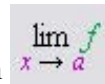
This section describes the key Maple calculus commands, many of which are used in task templates or available in the context menus.

For a complete list of calculus commands, refer to the **Mathematics** (including **Calculus**, **Differential Equations**, **Power Series**, and **Vector Calculus** subfolders) and **Student Package** sections of the Maple Help System Table of Contents.

### Limits

**To compute the limit of an expression as the independent variable approaches a value:**

1. In the **Expression** palette, click the limit item



2. Specify the independent variable, limit point, and expression, and then evaluate it.

For example:

$$> \lim_{x \rightarrow 0} \left( \frac{x}{\sin(x)} \right)$$

1

## The limit Command

By default, Maple searches for the real bidirectional limit (unless the limit point is  $\infty$  or  $-\infty$ ). To specify a direction, include one of the options **left**, **right**, **real**, or **complex** in a call to the **limit** command. See Table 4.9.

**Table 4.9: Limits**

Limit	Command Syntax	Output
$\lim_{x \rightarrow 0} \left( \frac{1}{x} \right)$	$> \text{limit} \left( \frac{1}{x}, x=0 \right)$	<i>undefined</i>
$\lim_{x \rightarrow 0^+} \left( \frac{1}{x} \right)$	$> \text{limit} \left( \frac{1}{x}, x=0, 'right' \right)$	$\infty$
$\lim_{x \rightarrow 0^-} \left( \frac{1}{x} \right)$	$> \text{limit} \left( \frac{1}{x}, x=0, 'left' \right)$	$-\infty$

Using the **limit** command, you can also compute multidimensional limits.

$$> \text{limit} \left( \frac{x^2}{y}, \{x=1, y=\infty\} \right)$$

0

For more information on multidimensional limits, refer to the **?limit/multi** help page.

## Numerically Computing a Limit

To numerically compute a limit:

- Use the **evalf(Limit(arguments))** calling sequence.

**Important:** Use the inert **Limit** command, not the **limit** command. For more information, refer to the **?limit** help page.

The **Limit** command accepts the same arguments as the **limit** command.

For example:

$$\begin{aligned} > \text{evalf}\left(\text{Limit}\left(\frac{\sin(x)}{\cos(x) + \tan(x)}, x = 1.225\right)\right) \\ & \qquad \qquad \qquad 0.3020605357 \end{aligned}$$

For information on the **evalf** command, see *Numerical Approximation* (page 313).

The **Limit** command does not compute the limit. It returns an unevaluated limit.



$$\begin{aligned} > \text{Limit}\left(\frac{\sin(x)}{\cos(x) + \tan(x)}, x = 1.225\right) \\ & \qquad \qquad \qquad \lim_{x \rightarrow 1.225} \frac{\sin(x)}{\cos(x) + \tan(x)} \end{aligned}$$

For more information on the **Limit** command, refer to the **?Limit** help page.

## Differentiation

Maple can perform symbolic and numeric differentiation.

**To differentiate an expression:**

1. In the **Expression** palette, click the differentiation item  or the partial differentiation item .

2. Specify the expression and independent variable, and then evaluate it.

For example, to differentiate  $x \sin(ax)$  with respect to  $x$ :

$$> \frac{d}{dx} (x \sin(ax))$$

$$\sin(ax) + x \cos(ax) a$$

You can also differentiate using context menus. For more information, see *Context Menus* (page 20).

To calculate a higher order or partial derivative, edit the derivative symbol inserted. For example, to calculate the second derivative of  $x \sin(ax) + x^2$  with respect to  $x$ :

$$> \frac{d^2}{dx^2} (x \sin(ax) + x^2)$$

$$2 \cos(ax) a - x \sin(ax) a^2 + 2$$

To calculate the mixed partial derivative of  $x \sin(3y) + yx^5$ :

$$> \frac{\partial^2}{\partial y \partial x} (x \sin(3y) + yx^5)$$

$$3 \cos(3y) + 5x^4$$

### The diff Command

Maple computes derivatives using the **diff** command. To directly use the **diff** command, specify the expression to differentiate and the variable.

>  $x \sin(ax) + x^2$

$$x \sin(ax) + x^2 \quad (4.1)$$

>  $\text{diff}((4.1), x)$

$$\sin(ax) + x \cos(ax) a + 2x$$

For information on equation labels such as (4.1), see *Equation Labels* (page 59).

To calculate a higher order derivative, specify a sequence of differentiation variables. Maple recursively calls the **diff** command.

>  $\text{diff}((4.1), x, x)$

$$2 \cos(ax) a - x \sin(ax) a^2 + 2$$

To calculate a partial derivative, use the same syntax. Maple assumes that the derivatives commute.

>  $\text{diff}(x \sin(3y) + y\sqrt{x}, x, y)$

$$3 \cos(3y) + \frac{1}{2\sqrt{x}}$$

To enter higher order derivatives, it is convenient to use the sequence operator (\$). For more information, refer to the ?\$ help page.

To compute the  $n^{\text{th}}$  derivative of an expression  $f$  in the independent variable  $t$ , you can use the syntax `diff(f, t$n)` .

For example:

```
> diff(cos(t), t$5)
```

–  $\sin(t)$

## Differentiating an Operator

You can also specify a mathematical function as a *functional operator* (a mapping). For a comparison of operators and other expressions, see *Distinction between Functional Operators and Other Expressions* (page 293).

### To find the derivative of a functional operator:

- Use the **D** operator.


The **D** operator returns a functional operator.

For example, find the derivative of an operator that represents the mathematical function  $x \cos(x)$  .

First, define the mathematical function  $x \cos(x)$  as the operator  $F$ .

1. In the **Expression** palette, click the single-variable function definition item  $f := x \rightarrow y$ .

2. Enter placeholder values.

- To move to the next placeholder, press the **Tab** key. **Note:** If pressing the **Tab** key inserts a tab, click the Tab icon  in the toolbar.

```
> F := x → x cos(x):
```

Now, define the operator,  $G$ , that maps  $x$  to the derivative of  $x \cos(x)$  .

>  $G := D(F)$

$$G := x \mapsto \cos(x) - x \sin(x)$$

$F$  and  $G$  evaluated at  $\pi/2$  return the expected values.

>  $F\left(\frac{\pi}{2}\right); G\left(\frac{\pi}{2}\right)$

0

$-\frac{1}{2}\pi$

For more information on the **D** operator, refer to the **?D** help page. For a comparison of the **diff** command and **D** operator, refer to the **?diffVersusD** help page.

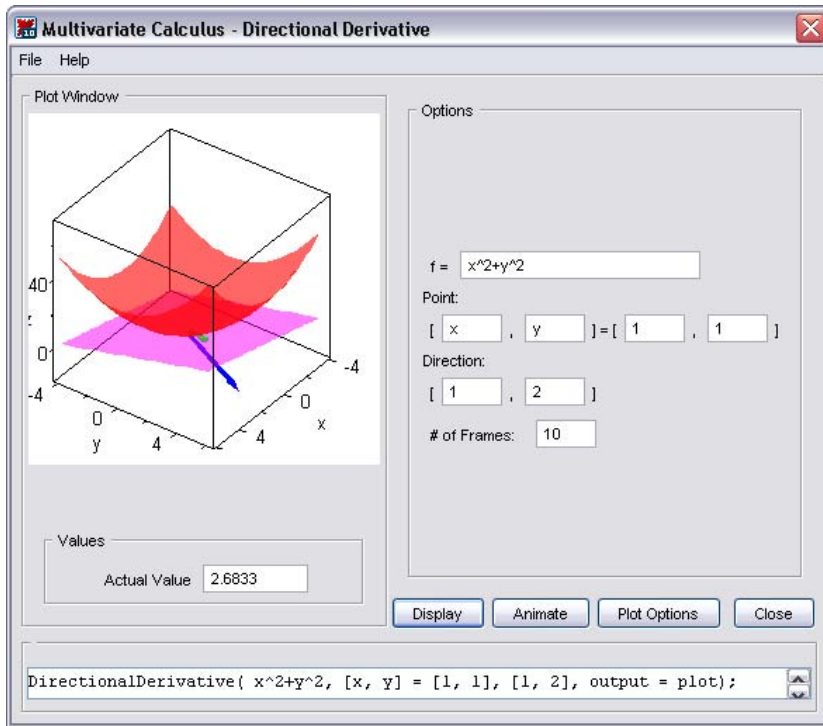
## Directional Derivative

To compute and plot a directional derivative, use the **Directional Derivative Tutor**. The tutor computes a floating-point value for the directional derivative.

### To launch the tutor:

From the **Tools** menu, select **Tutors, Calculus - Multi-Variable**, and then **Directional Derivatives**. Maple launches the **Directional Derivative Tutor**. See Figure 4.7.





**Figure 4.7: Directional Derivative Tutor**

To compute a symbolic value for the directional derivative, use the **Student[MultivariateCalculus][DirectionalDerivative]** command. The first list of numbers specifies the point at which to compute the derivative. The second list of numbers specifies the direction in which to compute the derivative.

For example, at the point  $[1, 2]$ , the gradient of  $x^2 + y^2$  points in the direction  $[2, 4]$ , which is the direction of greatest increase. The directional derivative in the orthogonal direction  $[-2, 1]$  is zero.

> *with(Student[MultivariateCalculus]):*

> *DirectionalDerivative( $x^2 + y^2$ , [x, y] = [1, 2], [1, 2]);*

$$2\sqrt{5}$$

> *DirectionalDerivative*( $x^2 + y^2$ ,  $[x, y] = [1, 2], [-2, 1]$ );

0

## Series

To generate the Taylor series expansion of a function about a point, use the **taylor** command.

> *taylor*( $\sin(4x)\cos(x), x=0$ )

$$4x - \frac{38}{3}x^3 + \frac{421}{30}x^5 + O(x^6)$$

**Note:** If a Taylor series does not exist, use the **series** command to find a general series expansion.

For example, the cosine integral function does not have a Taylor series expansion about 0. For more information, refer to the **?Ci** help page.

> *taylor*(*Ci*( $x$ ),  $x=0$ )

Error, does not have a Taylor expansion, try series()

To generate a truncated series expansion of a function about a point, use the **series** command.

> *series*(*Ci*( $x$ ),  $x=0$ )

$$\gamma + \ln(x) - \frac{1}{4}x^2 + \frac{1}{96}x^4 + O(x^6)$$

By default, Maple performs series calculations up to order 6. To use a different order, specify a non-negative integer third argument.

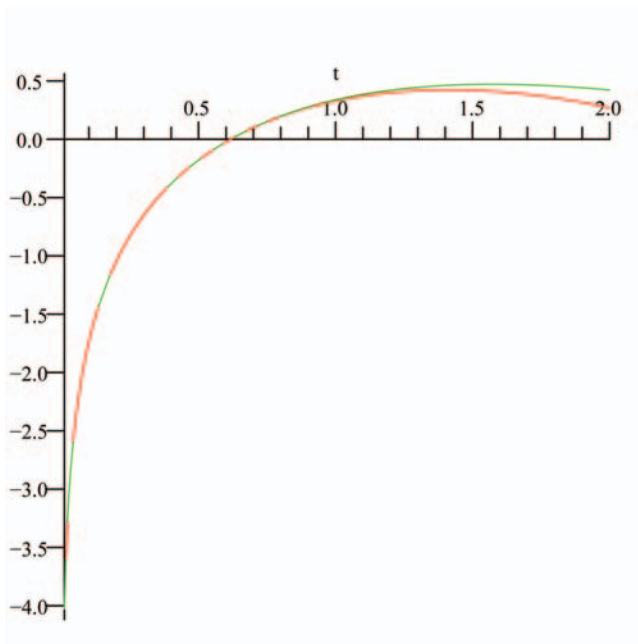
> `expansion := series(Ci(t), t = 0, 4)`

$$\text{expansion} := \gamma + \ln(t) - \frac{1}{4}t^2 + O(t^4)$$

To set the order for all computations, use the **Order** environment variable. For information about the **Order** variable and the  $O(t^4)$  term, refer to the **?Order** help page.

The expansion is of type **series**. Some commands, for example, **plot**, do not accept arguments of type **series**. To use the expansion, you must convert it to a polynomial using the **convert/polynom** command.

> `plot({Ci(t), convert(expansion, polynom)}, t = 1/100..2)`



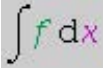
For information on Maple types and type conversions, see *Maple Expressions* (page 285).

For information on plotting, see *Plots and Animations (page 189)*.

## Integration

Maple can perform symbolic and numeric integration.

### To compute the indefinite integral of an expression:

1. In the **Expression** palette, click the indefinite integration item .
2. Specify the integrand and variable of integration, and then evaluate it.

For example, to integrate  $x \sin(ax)$  with respect to  $x$ :

$$> \int x \sin(ax) dx$$

$$\frac{\sin(ax) - x \cos(ax) a}{a^2}$$

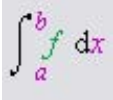
Recall that you can also enter symbols, including  $\int$  and  $d$ , using symbol completion.

- Enter the symbol name (or part of the name), for example, **int** and **d**, and then press the completion shortcut key.

For more information, see *Symbol Names (page 16)*.

You can also compute an indefinite integral using context menus. For more information, see *Context Menus (page 20)*.

### To compute the definite integral of an expression:

1. In the **Expression** palette, click the definite integration item .

2. Specify the endpoints of the interval of integration, integrand expression, and variable of integration, and then evaluate it.

For example, to integrate  $e^{(-a t)} \ln(t)$  over the interval  $(0, \infty)$ :

$$\begin{aligned} > \int_0^{\infty} e^{(-a t)} \ln(t) dt \\ & \lim_{t \rightarrow \infty} \left( -\frac{e^{-a t} \ln(t) + \text{Ei}_1(a t) + \gamma + \ln(a)}{a} \right) \end{aligned}$$

Maple treats the parameter **a** as a complex number. As described in *Assumptions on Variables* (page 117), you can compute under the assumption that **a** is a positive, real number using the **assuming** command.

$$\begin{aligned} > \int_0^{\infty} e^{-a t} \ln(t) dt \text{ assuming } a > 0 \\ & -\frac{\ln(a)}{a} - \frac{\gamma}{a} \end{aligned}$$

## The int Command

$\int f dx$  and  $\int_a^b f dx$  use the **int** command. To use the **int** command directly, specify the following arguments.

- Expression to integrate
- Variable of integration

$$> x \sin(ax)$$

$$x \sin(ax) \quad (4.2)$$

> `int((4.2),x)`

$$\frac{\sin(ax) - x \cos(ax) a}{a^2}$$

For a definite integration, set the variable of integration equal to the interval of integration.

> `int((4.2),x=0.. $\frac{\pi}{a}$ )`

$$\frac{\pi}{a^2}$$

## Numeric Integration

**To perform numeric integration:**

- Use the `evalf(Int(arguments))` calling sequence.

**Important:** Use the inert `Int` command, not the `int` command. For more information, refer to the `?int` help page.

In addition to the arguments accepted by the `int` command, you can include optional arguments such as `method`, which specifies the numeric integration method.

For example:

> `evalf(Int( $\frac{1}{\Gamma(x)}$ ,x=0..2,'method'=_Dexp))`

1.626378399

**Note:** To enter an underscore character (`_`) in 2-D Math, enter `\_`.

For information on the **evalf** command, see *Numerical Approximation* (page 313).

For information on numeric integration, including iterated integration and controlling the algorithm, refer to the **?evalf/Int** help page.

To compute iterated integrals, line integrals, and surface integrals, use the task templates (**Tools>Tasks>Browse**) in the **Multivariate** and **Vector Calculus** folders.

## Differential Equations

Maple has a powerful set of solvers for ordinary differential equations (ODEs) and partial differential equations (PDEs), and systems of ODEs and PDEs.

For information on solving ODEs and PDEs, see *Other Specialized Solvers* (page 88).

## Calculus Packages

In addition to top-level calculus commands, Maple contains calculus packages.

### VectorCalculus Package

The **VectorCalculus** package contains commands that perform multivariate and vector calculus operations on **VectorCalculus** vectors (vectors with an additional coordinate system attribute) and vector fields (vectors with additional coordinate system and **vectorfield** attributes), for example, **Curl**, **Flux**, and **Torsion**.

> *with(VectorCalculus):*

> *SetCoordinates('cartesian[x,y,z]):*

> *VectorField1 := VectorField(< -y,x, z>)*

$$\text{VectorField1} := \begin{bmatrix} -y \\ x \\ z \end{bmatrix}$$

Find the curl of **VectorField1**.

> *Curl(VectorField1);*

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

Find the flux of **VectorField1** through a sphere of radius **r** at the origin.

> *Flux(VectorField1, Sphere(< 0, 0, 0 >, r))*

$$\frac{4}{3} r^3 \pi$$

Compute the torsion of a space curve. The curve must be a vector with parametric function components.

> *simplify(Torsion(< t, t<sup>2</sup>, t<sup>3</sup> >, t)) assuming t::real*

$$3 \frac{1}{9t^4 + 9t^2 + 1}$$

For information on the **assuming** command, see *The assuming Command* (page 119).

For more information on the **VectorCalculus** package, including a complete list of commands, refer to the **?VectorCalculus** help page.



To find other calculus packages, such as **VariationalCalculus**, refer to the **?index/package** help page.

## Student Calculus Packages

The **Student** package contains subpackages that help instructors teach concepts and allow students to visualize and explore ideas. These subpackages also contain computational commands. The **Student** calculus subpackages include **Calculus1**, **MultivariateCalculus**, and **VectorCalculus**. The **Student[VectorCalculus]** package provides a simple interface to a limited subset of the functionality available in the **VectorCalculus** package.

For information on using Maple as a teaching and learning tool, and some computational examples, see *Teaching and Learning with Maple* (page 180).

## 4.5 Optimization

Using the **Optimization** package, you can numerically solve optimization problems. The package uses fast Numerical Algorithms Group (NAG) algorithms to *minimize* or *maximize* an objective function.

The **Optimization** package solves constrained and unconstrained problems.

- Linear programs
- Quadratic programs
- Nonlinear programs
- Linear and nonlinear least-squares problems

The **Optimization** package contains local solvers. In addition, for univariate finitely-bounded nonlinear programs with no other constraints, you can compute global solutions using the **NLPSolve** command. To find global solutions generally, purchase the **Global Optimization Toolbox**. For more information, visit <http://www.maplesoft.com/products/toolboxes>.

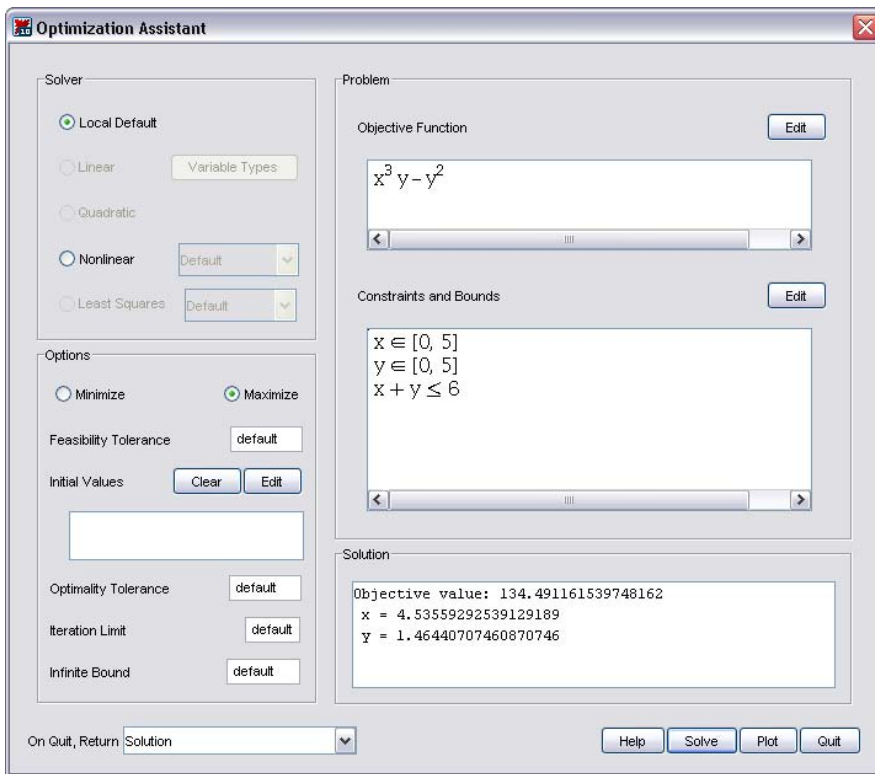
## Point-and-Click Interface

The primary method for solving optimization problems is the **Optimization Assistant**.

**To launch the Optimization Assistant:**

- From the **Tools** menu, select **Assistants**, and then **Optimization**.

Maple inserts the `Optimization[Interactive]()` calling sequence (in Worksheet mode), and launches the **Optimization Assistant**. See Figure 4.8.



**Figure 4.8: Optimization Assistant**

To solve a problem:

1. Enter the objective function, constraints, and bounds.

2. Select the **Minimize** or **Maximize** radio button.
3. Click the **Solve** button. The solution is displayed in the **Solution** text box.

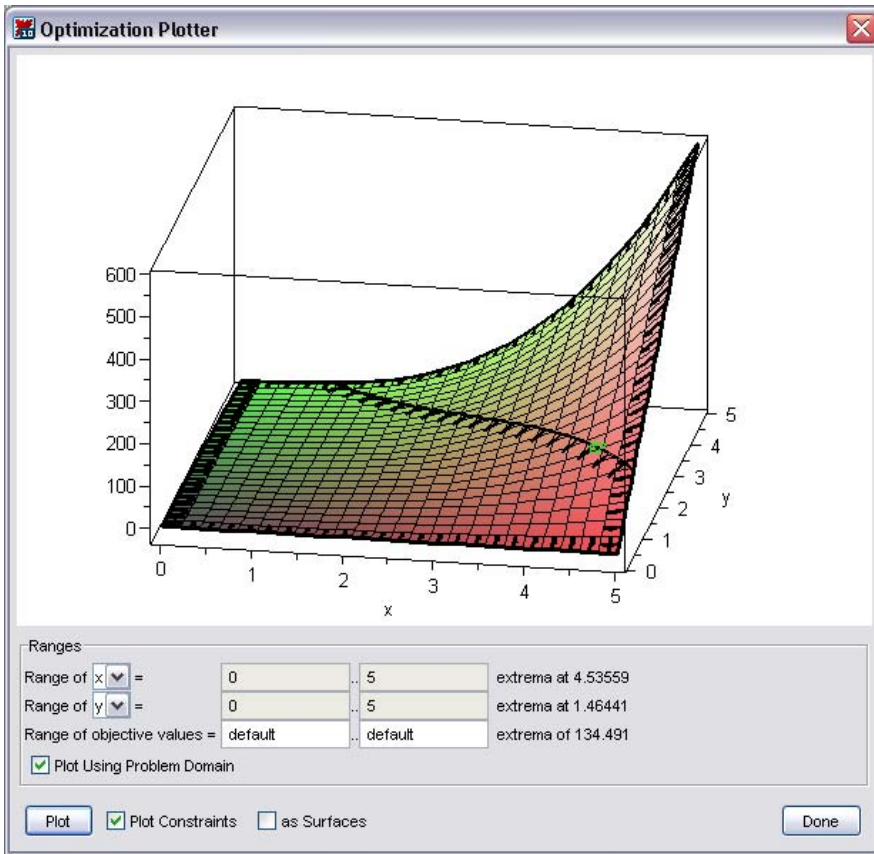
You can also enter the problem (objective function, constraints, and bounds) in the calling sequence.

For example, find the maximum of  $x^3y - y^2$  subject to the constraints  $x + y \leq 6, x \in [0,5], y \in [0,5]$ .

```
> Optimization[Interactive](x3y - y2, {x + y ≤ 6, x = 0..5, y = 0..5})  
  
[  
  134.491161539748276  
  , [y = 1.46440707460870768,  
    x = 4.53559292539129277]]
```

After finding a solution, you can plot it. To plot a solution:

In the **Optimization Assistant** window, click the **Plot** button. The **Optimization Plotter** window is displayed. See Figure 4.9.



**Figure 4.9: Optimization Assistant Plotter Window**

For information on the algorithms used to solve optimization problems, refer to the **?Optimization/Methods** help page.

## Large Optimization Problems

The **Optimization Assistant** accepts input in an algebraic form. You can specify input in other forms, described in the **?Optimization/InputForms** help page, in command calling sequences.

The Matrix form, described in the **?Optimization/MatrixForm** help page, is more complex but offers greater flexibility and efficiency.

For example, solve the quadratic program:

maximize  $c^T x + \frac{1}{2} x^T H x$  subject to  $Ax \leq b$ , where  $x$  is the vector of problem variables.

Define the column vector,  $\mathbf{c}$ , of the quadratic objective function.

```
> c := Vector([2, 5], 'datatype' = 'float');
```

Define the symmetric Hessian matrix,  $\mathbf{H}$ , of the quadratic objective function.

```
> H := Matrix([[6, 3], [3, 4]], 'datatype' = 'float');
```

Define the matrix  $\mathbf{A}$ , the coefficient matrix for the linear inequality constraints.

```
> A := Matrix([-1, 1], 'datatype' = 'float');
```

Define the column vector  $\mathbf{b}$ , the linear inequality constraints.

```
> b := Vector([-2], 'datatype' = 'float');
```

The **QPSolve** command solves quadratic programs.

```
> Optimization[QPSolve]([c, H], [A, b])
```

```
[-3.53333333333333, [0.466666666666666564  
-1.600000000000000030]]
```

**Note:** For information on creating matrices and vectors (including how to use the **Matrix** palette to easily create matrices), see *Linear Algebra* (page 135).

For additional information on performing efficient computations, refer to the **?Optimization/Computation** help page.

## MPS(X) File Support

To import linear programs from a standard MPS(X) data file, use the **Import-MPS** command.

## Additional Information

For a complete list of commands and other **Optimization** package information, refer to the **?Optimization** help page.

# 4.6 Statistics

The **Statistics** package is a collection of commands and the point-and-click **Data Analysis Assistant**—refer to the **?Statistics[InteractiveDataAnalysis]** help page—for performing computations in mathematical statistics and data analysis. The package supports a wide range of common statistical tasks including quantitative and graphical data analysis, simulation, and curve fitting.

In addition to standard data analysis tools, the **Statistics** package provides a wide range of symbolic and numeric tools for computing with random variables. The package supports over 35 major probability distributions and can be extended to include new distributions.

## Probability Distributions and Random Variables

The **Statistics** package supports:

- Continuous distributions, which are defined along the real line by probability density functions. Maple supports many continuous distributions, including the normal, Student-t, Laplace, and logistic distributions.
- Discrete distributions, which have nonzero probability only at discrete points. A discrete distribution is defined by a probability function. Maple supports many discrete distributions, including the Bernoulli, geometric, and Poisson distributions.

For a complete list of distributions, refer to the **?Statistics/Distributions** help page.

You can define random variables by specifying a distribution in a call to the **RandomVariable** command.

> *with(Statistics):*

> *X := RandomVariable(Poisson( $\lambda$ )):*

Find the probability distribution function for **X**. (For information on statistics computations, see *Statistical Computations* (page 175)).

> *PDF(X, t)*

$$\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda} \delta(t-k)}{k!}$$

$\delta$  represents the Dirac delta function. For more information, refer to the **?Dirac** help page.

### Adding Custom Distributions

To add a new distribution, specify a probability distribution in a call to the **Distribution** command.

> *U := Distribution*  $\left( PDF = \left( t \rightarrow \begin{cases} 0 & t < 0 \\ \frac{1}{3} & 0 \leq t < 3 \\ 0 & \text{otherwise} \end{cases} \right) \right)$  :

To construct a piecewise-continuous function in 1-D Math, use the **piecewise** command, for example, **t -> piecewise(t < 0, 0, t < 3, 1/3, 0)**.

Define a new random variable with this distribution.

> *Z := RandomVariable(U): PDF(Z, t)*

$$\begin{cases} 0 & t < 0 \\ \frac{1}{3} & 0 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the mean value of the random variable.

> *Mean(Z)*

$$\frac{3}{2}$$

For more information, refer to the [?Statistics/Distributions](#) help page.

## Statistical Computations

In addition to basic functions, like mean, median, standard deviation, and percentile, the **Statistics** package contains commands that compute, for example, the interquartile range and hazard rate.

### Examples

#### Example 1

Compute the average absolute range from the interquartile of the Rayleigh distribution with scale parameter 3.

> *InterquartileRange(Rayleigh(3))*

$$\sqrt{36} \sqrt{\ln(2)} - \sqrt{-18 \ln\left(\frac{3}{4}\right)}$$

To compute the result numerically:

- Specify the '**numeric**' option.



> *InterquartileRange*(*Rayleigh*(3), 'numeric')

2.719744818

## Example 2

Compute the hazard rate of the Cauchy distribution with location and scale parameters **a** and **b** at an arbitrary point **t**.

> *HazardRate*(*Cauchy*(*a*, *b*), *t*)

$$\frac{1}{\pi b \left( 1 + \frac{(t-a)^2}{b^2} \right) \left( \frac{1}{2} - \frac{\arctan\left(\frac{t-a}{b}\right)}{\pi} \right)}$$

You can specify a value for the point **t**.

> *HazardRate*(*Cauchy*(*a*, *b*),  $\frac{1}{2}$ )

$$\frac{1}{\pi b \left( 1 + \frac{\left(\frac{1}{2} - a\right)^2}{b^2} \right) \left( \frac{1}{2} - \frac{\arctan\left(\frac{\frac{1}{2} - a}{b}\right)}{\pi} \right)}$$

You can also specify that Maple compute the result numerically.

> *HazardRate*(*Cauchy*(10, 1),  $\frac{1}{2}$ , 'numeric')

0.003608801460

For more information, refer to the **?Statistics/DescriptiveStatistics** help page.

## Plotting

You can generate statistical plots using the visualization commands in the **Statistics** package. Available plots include:

- Bar chart
- Frequency plot
- Histogram
- Pie Chart
- Scatter Plot

For example, create a scatter plot for a distribution of points that vary from

$\sin\left(\frac{2\pi x}{200}\right)$  by a small value determined by a normally distributed sample.

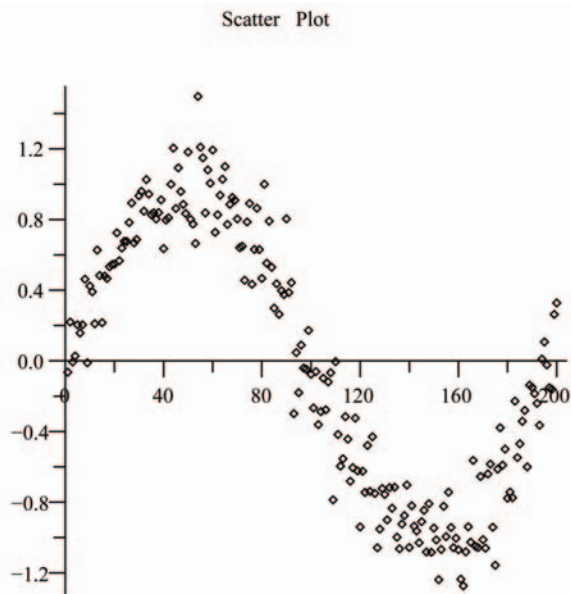
```
> N := 200;
```

```
> U := Sample(Normal(0, 1), N);
```

```
> X := <seq(x, x = 1 .. N)>:
```

```
> Y := <seq\left(\sin\left(\frac{2\pi x}{N}\right) + \frac{U[x]}{5}, x = 1 .. N\right)>:
```

```
> ScatterPlot(X, Y, 'title' = "Scatter Plot");
```



For information on plotting options, such as **title**, see *Plots and Animations* (page 189).

To fit a curve to the data points, include the optional **fit** equation parameter.

Using the **plots[display]** command, create a plot that contains the:

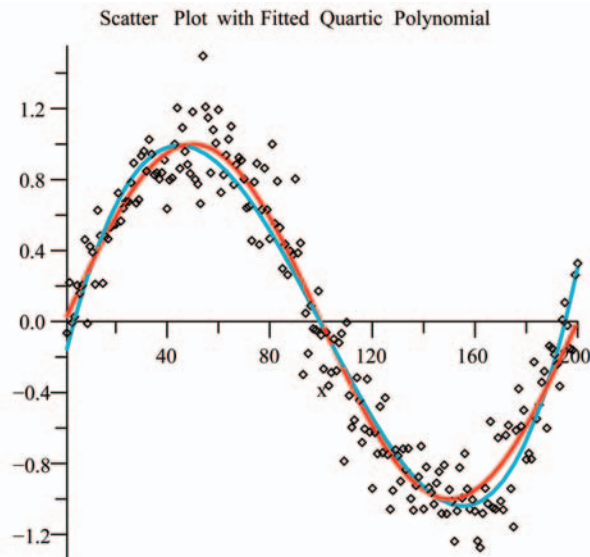
- Scatter plot of the data points
- Quartic polynomial fitted to the data points:

$$f(x) = ax^4 + bx^3 + cx^2 + dx + e$$

- Function  $\sin(2\pi x/N)$

```
> P := ScatterPlot(X, Y, fit = [ax^4 + bx^3 + cx^2 + dx + e, x],
  thickness = 2):
```

- ```
> Q:=plot(sin(2*pi*x/N),x=1..N,thickness=2,color=red,  
          linestyle=DOT):  
  
> plots[display](P,Q,'title'=  
  "Scatter Plot with Fitted Quartic Polynomial")
```



For more information on statistical plots, refer to the [?Statistics/Visualization](#) help page.

For an overview of plotting, see *Plots and Animations* (page 189).

## Additional Information

For more information on the **Statistics** package, including regression analysis, estimation, data manipulation, and data smoothing, refer to the [?Statistics](#) help page.

## 4.7 Teaching and Learning with Maple

Table 4.10 resources for instructors and students. For additional resources see *Table 4.1 (page 123)*.

**Table 4.10: Student and Instructor Resources**

| Resource                               | Description                                                                                                                                                                                                                                                                                                                          |
|----------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Student Packages and Tutors            | The <b>Student</b> package contains computational and visualization (plotting and animation) functionality, and point-and-click interfaces for explaining and exploring concepts ( <b>Tools&gt;Tutors</b> ). For more information, refer to the <b>?Student</b> help page.                                                           |
| Mathematics and Engineering Dictionary | The Maple Help System has an integrated dictionary of over 5000 mathematics and engineering terms. You can search the dictionary using the Help System search engine. ( <b>Help&gt;Manuals, Dictionary, and more&gt;Dictionary</b> )                                                                                                 |
| Maple Application Center <sup>TM</sup> | The Maple Application Center contains tutorials and applications that help instructors begin using Maple and use Maple in the classroom. Browse the many resources in the <b>Education</b> and <b>Education PowerTools</b> categories. ( <a href="http://www.maplesoft.com/applications">http://www.maplesoft.com/applications</a> ) |

| Resource              | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|-----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Maple Student Center™ | <p>The Maple Student Center contains tutorials and applications that help students learn how to use Maple, explore mathematical concepts, and solve problems. Available resources include:</p> <ul style="list-style-type: none"> <li>• Study guides - Complete lessons with examples for academic courses, including precalculus and calculus. For example, the Interactive Precalculus Study Guide contains worked problems, each solved as in a standard textbook, using Maple commands and custom Maplet graphical interfaces.</li> <li>• Free course lessons for many subjects including precalculus to vector calculus; high school, abstract, and linear algebra; engineering; physics; differential equations; cryptography; and classical mechanics.</li> </ul> <p>(<a href="http://www.maplesoft.com/academic/students">http://www.maplesoft.com/academic/students</a>)</p> |

## Student Packages and Tutors

The **Student** package is a collection of subpackages for teaching and learning mathematics and related subjects. The **Student** package contains packages for a variety of subjects, including precalculus, calculus, and linear algebra.

Instructors can:

- Teach concepts without being distracted by the mechanics of the computations.
- Create examples and quickly update them during a lesson to demonstrate different cases or show the effect of the variation of a parameter.
- Create plots and animations to visually explain concepts, for example, the geometric relationship between a mathematical function and its derivatives (**Tools>Tutors>Calculus - Single Variable>Derivatives**). See Figure 4.10.

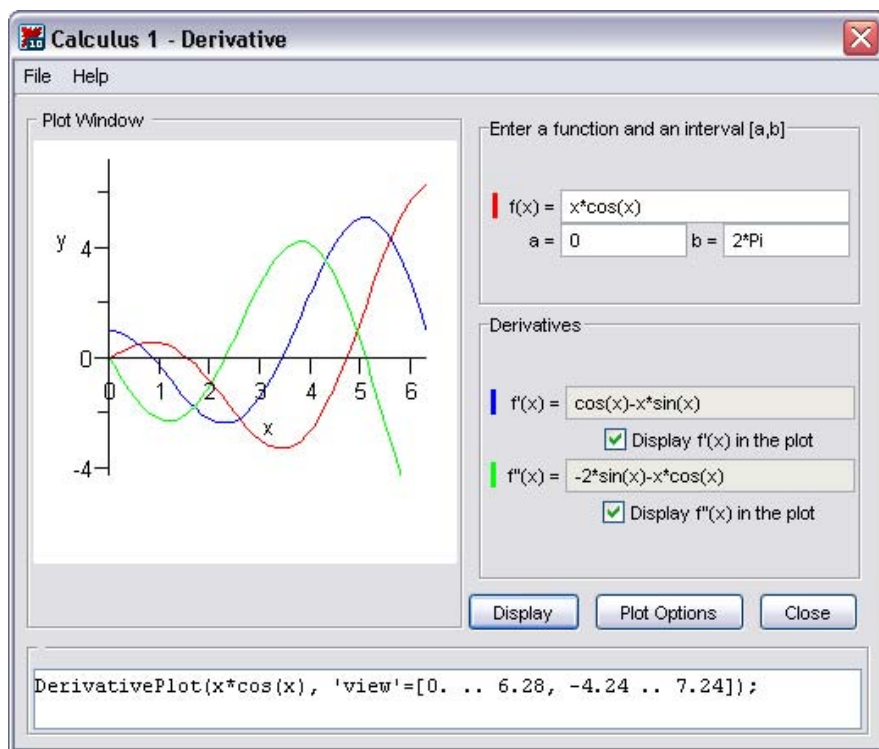
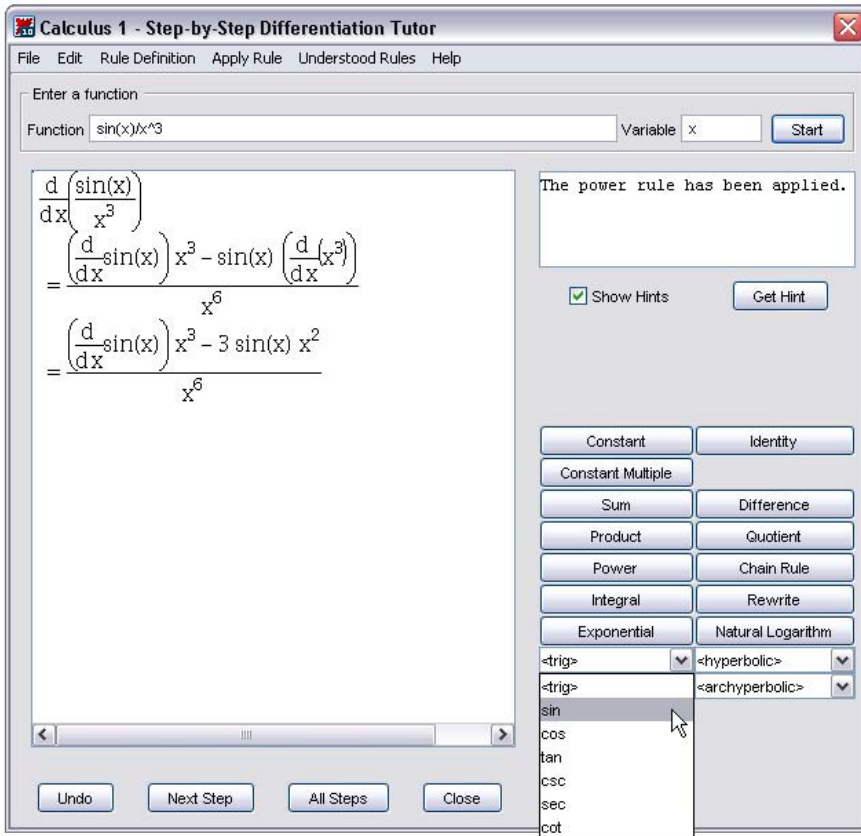


Figure 4.10: Student[Calculus1] Derivatives Tutor

Students can:

- Perform step-by-step computations, for example, compute a derivative by applying differentiation rules using commands or a tutor (**Tools>Tutors>Calculus - Single Variable>Differentiation Methods**). See Figure 4.11.
- Perform computations.
- Visually explore concepts.



**Figure 4.11: Student[Calculus1] Differentiation Methods Tutor**

Tutors provide point-and-click interfaces to the **Student** package functionality.

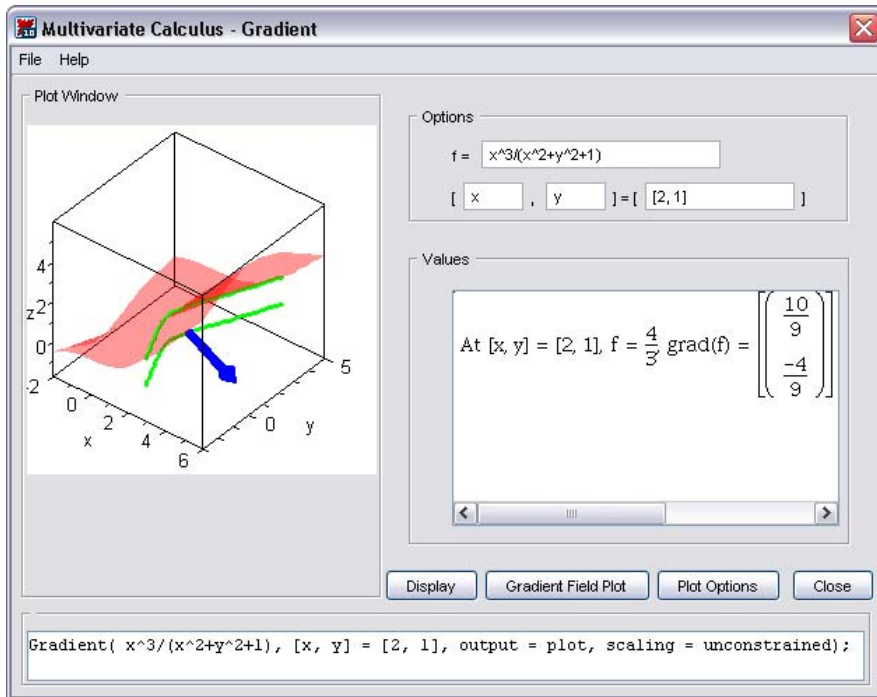
**To launch a tutor:**

1. From the **Tools** menu, select **Tutors**.
2. Select a subject, for example, **Calculus - Multi-Variable**.
3. Select a tutor, for example, **Gradients**.

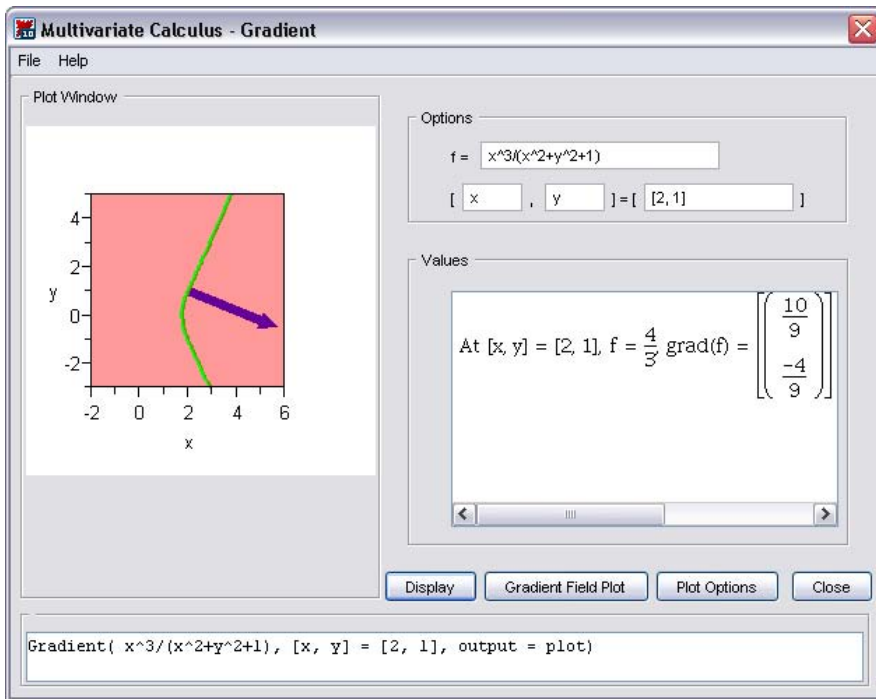


Maple inserts the `Student[MultivariateCalculus][GradientTutor]()` calling sequence (in Worksheet mode), and launches the **Multivariate Calculus Gradient Tutor**.

By rotating the three-dimensional plot, you can show that the gradient points in the direction of greatest increase of the surface (see Figure 4.12) and show the direction of the gradient vector in the x-y plane (see Figure 4.13).



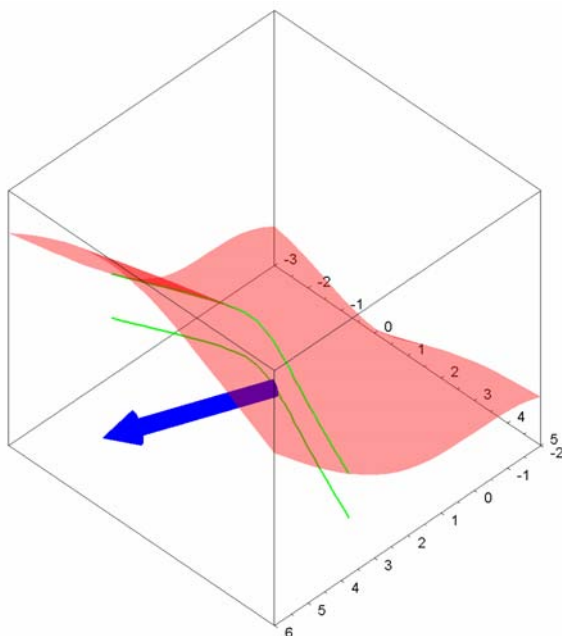
**Figure 4.12: Multivariate Calculus Gradient Tutor**



**Figure 4.13: Multivariate Calculus Gradient Tutor Showing x-y Plane**

When you close the tutor, Maple inserts the 3-D plot.

> `Student[MultivariateCalculus][GradientTutor]();`



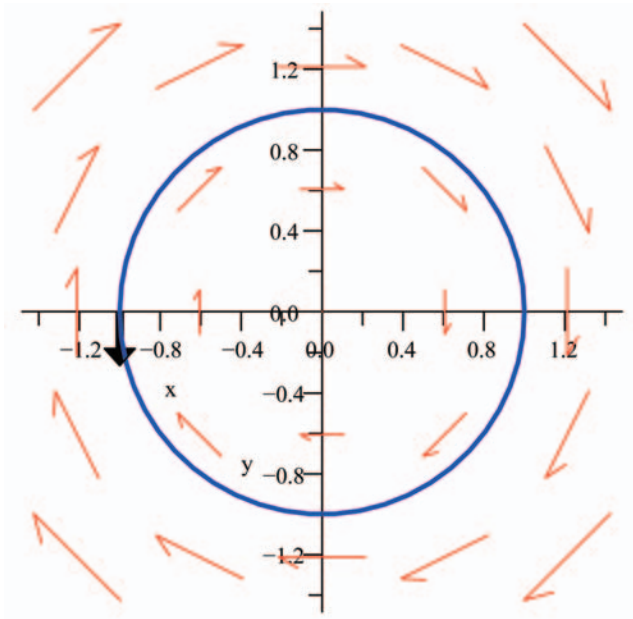
Many **Student** package commands can return a value, mathematical expression, plot, or animation. This allows you to compute the final answer, see the general formula applied to a specific problem, or visualize the underlying concepts.

For example, the **Student[VectorCalculus][LineInt]** (line integral) command can return the following.

- Plot that visually indicates the vector field, path of integration, and tangent vectors to the path
- Unevaluated line integral
- Numeric value of the line integral

> *with(Student[VectorCalculus]):*

> *LineInt(*  
*VectorField(< y, -x>), Circle(< 0, 0>, 1), 'output' = 'plot')*



```
> LineInt(
  VectorField(< y, -x>),
  Circle(< 0, 0>, 1), 'output'='integral')
```

$$\int_0^{2\pi} (-\sin(t)^2 - \cos(t)^2) dt \quad (4.3)$$

To evaluate the integral returned by the **output = integral** calling sequence, use the **value** command.

```
> value((4.3))
```

$$-2\pi$$

By default, the **LineInt** command returns the value of the integral.

```
> LineInt(VectorField(< y - x, -x - y>), Circle(< 0, 0>, r))
```

$$-2\pi r^2$$

For more information on the **Student** package, refer to the **?Student** help page.

# 5 Plots and Animations

Maple can generate many forms of plots, allowing you to visualize a problem and further understand concepts.

- Maple accepts explicit, implicit, and parametric forms to display 2-D and 3-D plots and animations.
- Maple recognizes many coordinate systems.
- All plot regions in Maple are active; therefore, you can drag expressions to and from a plot region.
- Maple offers numerous plot options, such as axes styles, title, colors, shading options, surface styles, and axes ranges, which give you complete control to customize your plots.

## 5.1 In This Chapter

| Section                                                                                | Topics                                                                                                                                                                                                                                                                                |
|----------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Creating Plots - Interactive and command-driven methods to display 2-D and 3-D plots   | <ul style="list-style-type: none"><li>• <b>Interactive Plot Builder</b></li><li>• Context Menu</li><li>• Dragging to a Plot Region</li><li>• The <b>plot</b> and <b>plot3d</b> Commands</li><li>• The <b>plots</b> Package</li><li>• Multiple Plots in the Same Plot Region</li></ul> |
| Customizing Plots - Methods for applying plot options before and after a plot displays | <ul style="list-style-type: none"><li>• <b>Interactive Plot Builder</b> Options</li><li>• Context Menu Options</li><li>• The <b>plot</b> and <b>plot3d</b> Command Options</li></ul>                                                                                                  |

| Section                                                                                           | Topics                                                                                                                                                                          |
|---------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Analyzing Plots - Plot analyzing tools                                                            | <ul style="list-style-type: none"><li>• Point Probe</li><li>• Rotate</li><li>• Pan</li><li>• Scale</li></ul>                                                                    |
| Creating Animations - Interactive and command-driven methods to display animations                | <ul style="list-style-type: none"><li>• <b>Interactive Plot Builder</b></li><li>• The <b>plots[animate]</b> Command</li></ul>                                                   |
| Playing Animations - Tools to run animations                                                      | <ul style="list-style-type: none"><li>• Animation Context Bar</li></ul>                                                                                                         |
| Customizing Animations - Methods for applying plot options before and after an animation displays | <ul style="list-style-type: none"><li>• <b>Interactive Plot Builder</b> Animation Options</li><li>• Context Menu Options</li><li>• The <b>animate</b> Command Options</li></ul> |
| Exporting - Methods for exporting plots                                                           | <ul style="list-style-type: none"><li>• Saving Plots to File Formats</li></ul>                                                                                                  |
| Code for Color Plates - Information on color plates                                               | <ul style="list-style-type: none"><li>• Accessing Code for the Color Plates</li></ul>                                                                                           |

## 5.2 Creating Plots

Maple offers several methods to easily plot an expression. These methods include:

- The **Interactive Plot Builder**
- Context menus
- Dragging to a plot region
- Commands

Each method offers a unique set of advantages. The method you use depends on the type of plot to display, as well as your personal preferences.

## Interactive Plot Builder

The **Interactive Plot Builder** is a point-and-click interface to the Maple plotting functionality. The interface displays plot types based on the expression you specify. The available plot types include plots, interactive plots, animations, or interactive animations. Depending on the plot type you select, you can create a:

- 2-D / 3-D plot
- 2-D / 3-D conformal plot of a complex-valued function
- 2-D / 3-D complex plot
- 2-D density plot
- 2-D gradient vector-field plot
- 2-D implicit plot

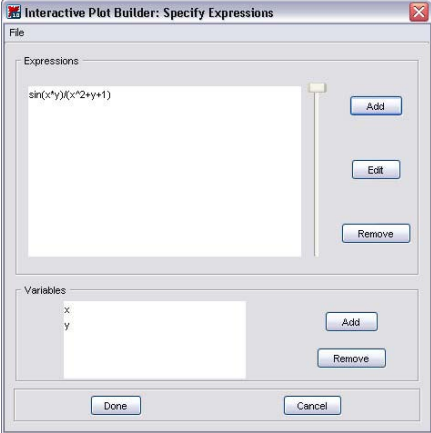
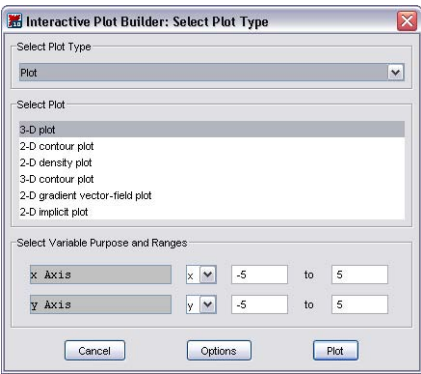
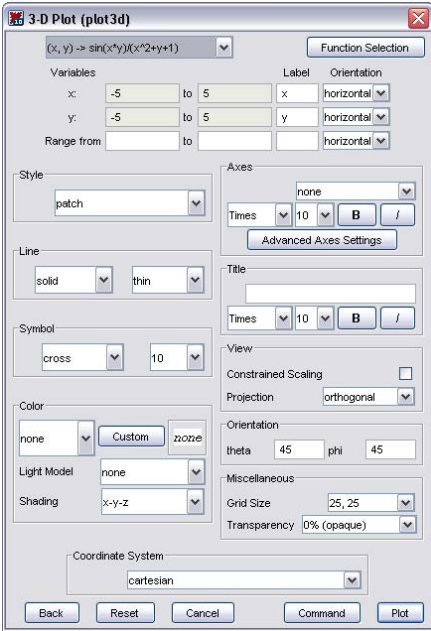
Using the **Interactive Plot Builder**, you can customize and display a plot by selecting from the numerous plot types and applying plot options without any knowledge of plotting command syntax. The output from the **Interactive Plot Builder** is a plot of the expression or the command used to generate the plot in the document.

To launch the **Interactive Plot Builder**:

- From the **Tools** menu, select **Assistants**, and then **Plot Builder**. **Note:** The **Tools** menu also offers tutors to easily generate plots in several academic subjects. For more information, see *Teaching and Learning with Maple* (page 180).



**Table 5.1: Windows of the Interactive Plot Builder**

|                                                                                                                               |                                                                                                                             |
|-------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| <p><b>1. Specify Expressions window</b></p>  | <p><b>2. Select Plot Type window</b></p>  |
| <p><b>3. Plot Options window</b></p>        |                                                                                                                             |

- **Specify Expressions window** - Add, edit, or remove expressions and variables. Once finished, you can advance to the **Select Plot Type** window.
- **Select Plot Type window** - Select the plot type and corresponding plot, and edit the ranges. Once finished, you can display the plot or advance to the **Plot Options** window.
- **Plot Options window** - Apply plot options. Once finished, you can display the plot or return the command that generates the plot to the document.

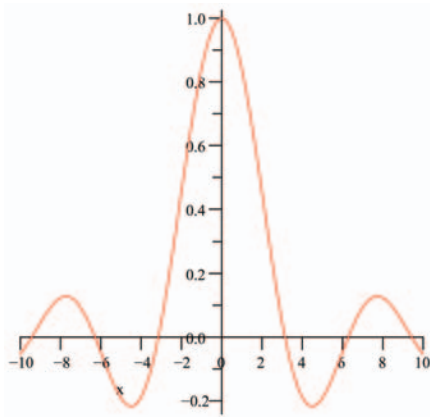
### Example 1 - Display a plot of a single variable expression

Maple can display two-dimensional graphs and offers numerous plot options such as color, title, and axes styles to customize the plot.

**Table 5.2: Displaying a Plot of a Single Variable Expression**

| Step                                         | Details                                                                                                                                                                                                                                                                                                                                                                                                               |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Launch the <b>Interactive Plot Builder</b> . | <p>1. Ensure the cursor is in a Maple input region.</p> <p>2. From the <b>Tools</b> menu, select <b>Assistants</b>, and then <b>Plot Builder</b>.</p> <p>Notes: 1. Maple inserts <code>plots[interactive]();</code> in the Maple document. Entering this command at the Maple prompt also invokes the Plot Builder.</p> <p>2. Interaction with the document is disabled while the <b>Plot Builder</b> is running.</p> |

| Step                 | Details                                                                                                                                                                                                                                                                                        |
|----------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Enter an expression. | <ol style="list-style-type: none"><li>1. In the <b>Specify Expressions</b> window:<ol style="list-style-type: none"><li>a. Add the expression, <b>sin(x)/x</b>.</li><li>b. Click <b>Done</b> to proceed to the <b>Select Plot Type</b> window.</li></ol></li></ol>                             |
| Plot the expression. | <ol style="list-style-type: none"><li>1. In the <b>Select Plot Type</b> window, notice the default setting of a 2-D plot type and an <b>x</b> axis range, <b>-10 .. 10</b>.<br/><br/>Notice also the various plot types available for this expression.</li><li>2. Click <b>Plot</b>.</li></ol> |



To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder (page 208)*

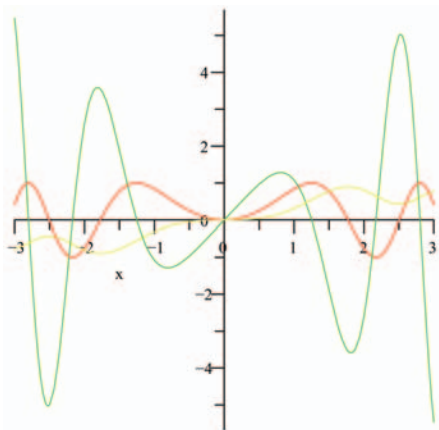
## Example 2 - Display a plot of multiple expressions of 1 variable

Maple can display multiple expressions in the same plot region to compare and contrast. The **Interactive Plot Builder** accepts multiple expressions.

**Table 5.3: Displaying a Plot of Multiple Expressions of 1 Variable**

| Step                                                                                             | Details                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|--------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Launch the <b>Interactive Plot Builder</b> and enter the expressions.                            | <ol style="list-style-type: none"> <li>1. Launch the <b>Interactive Plot Builder</b>.</li> </ol> <p>The <b>Plot Builder</b> accepts expressions and performs basic calculations on expressions. For example, entering <b>diff(sin(x<sup>2</sup>), x)</b> in the <b>Specify Expression</b> window performs the calculation and displays the expression as <b>2*cos(x<sup>2</sup>)*x</b> in the <b>Expression</b> group box.</p> <ol style="list-style-type: none"> <li>2. In the <b>Specify Expressions</b> window:               <ol style="list-style-type: none"> <li>a. In three separate steps, add the expressions <b>sin(x<sup>2</sup>)</b>, <b>diff(sin(x<sup>2</sup>),x)</b>, and <b>int(sin(x<sup>2</sup>), x)</b>.</li> </ol> </li> </ol> |
| Change the x-axis range.                                                                         | In the <b>Select Plot Type</b> window: <ol style="list-style-type: none"> <li>a. Change the <b>x Axis</b> range to <b>-3 .. 3</b>.</li> <li>b. Click <b>Options</b> to proceed to the <b>Plot Options</b> window.</li> </ol>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| Launch the <b>Plot Options</b> window and return the <b>plot</b> command syntax to the document. | Click <b>Command</b> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

| Step                     | Details                                                  |
|--------------------------|----------------------------------------------------------|
| Display the actual plot. | Execute the inserted command, that is, display the plot. |



By default, Maple displays each plot in a plot region using a different color. You can also apply a line style such as solid, dashed, or dotted for each expression in the graph. For more information, refer to the **?plot/options** help page. To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder* (page 208)

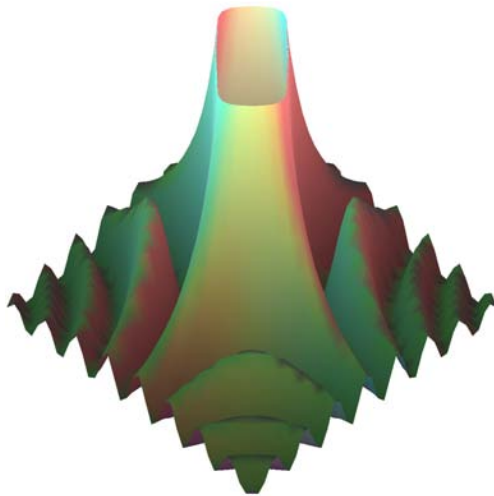
### Example 3 - Display a plot of a multi-variable expression

Maple can display three-dimensional plots and offers numerous plot options such as light models, surface styles, and shadings to allow you to customize the plot.

**Table 5.4: Displaying a Plot of a Multi-variable Expression**

| Step                                                                | Details                                        |
|---------------------------------------------------------------------|------------------------------------------------|
| Launch the <b>Interactive Plot Builder</b> and enter an expression. | Add the expression $(1+\sin(x*y))/(x^2+y^2)$ . |

| Step                                   | Details                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|----------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Launch the <b>Plot Options</b> window. | In the <b>Select Plot Type</b> window:<br><br>a. Notice the available plot types for an expression with 2 variables, as well as the plot objects for each type.<br><br>b. Click <b>Options</b> .                                                                                                                                                                                                                                                                                                                                                                                    |
| Set plot options.                      | In the <b>Plot Options</b> window:<br><br>a. From the <b>Variables</b> column, change the <b>Range</b> fields to <b>0 .. 0.05</b> .<br><br>b. From the <b>Labels</b> column, enter <b>z</b> .<br><br>c. From the <b>Color</b> group box, select <b>Light Model</b> , and then <b>green-red</b> .<br><br>d. From the <b>Color</b> group box, select <b>Shading</b> , and then <b>z (grayscale)</b> .<br><br>e. From the <b>Style</b> group box, select <b>patch w/o grid</b> .<br><br>f. From the <b>Miscellaneous</b> group box, select <b>Grid Size</b> , and then <b>40, 40</b> . |
| Plot the expression.                   | Click <b>Plot</b> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |



To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder* (page 208)

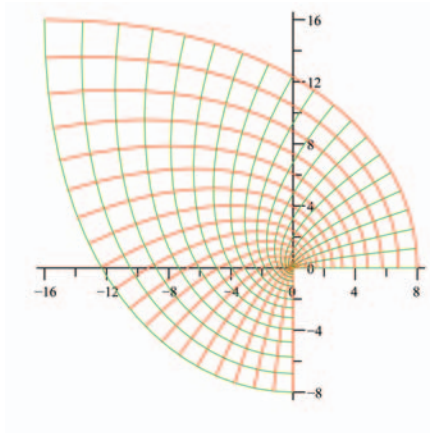
#### **Example 4 - Display a conformal plot**

Maple can display a conformal plot of a complex expression mapped onto a two-dimensional grid or plotted on the Riemann sphere in 3-D.

**Table 5.5: Displaying a Conformal Plot**

| Step                                                                | Details                    |
|---------------------------------------------------------------------|----------------------------|
| Launch the <b>Interactive Plot Builder</b> and enter an expression. | Add the expression $z^3$ . |

| Step                 | Details                                                                                                                                                                                                                                                                          |
|----------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Select a plot type.  | In the <b>Select Plot Type</b> window: <ol style="list-style-type: none"> <li>a. From the <b>Select Plot</b> group box, select <b>2-D conformal plot of a complex-valued expression</b>.</li> <li>b. Change the range of the <b>z</b> parameter to <b>0 .. 2+2*I</b>.</li> </ol> |
| Set plot options.    | In the <b>Plot Options</b> window: <ol style="list-style-type: none"> <li>a. From the <b>Axes</b> group box, select <b>normal</b>.</li> <li>b. From the <b>Miscellaneous</b> group box, select the <b>Grid Size</b> drop-down menu option <b>30, 30</b>.</li> </ol>              |
| Plot the expression. | Click <b>Plot</b> .                                                                                                                                                                                                                                                              |



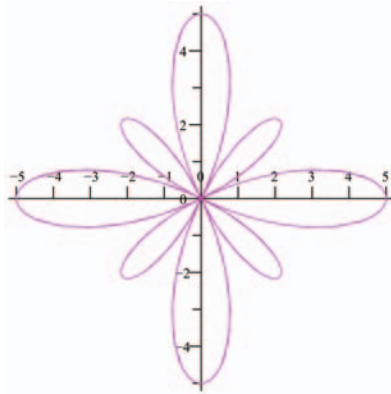


### Example 5 - Display a plot in polar coordinates

Cartesian (ordinary) coordinates is the Maple default. Maple also supports numerous other coordinate systems, including hyperbolic, inverse elliptic, logarithmic, parabolic, polar, and rose in two-dimensions, and bipolar cylindrical, bispherical, cylindrical, inverse elliptical cylindrical, logarithmic cosh cylindrical, Maxwell cylindrical, tangent sphere, and toroidal in three-dimensional plots. For a complete list of supported coordinate systems, refer to the **?coords** help page.

**Table 5.6: Displaying a Plot in Polar Coordinates**

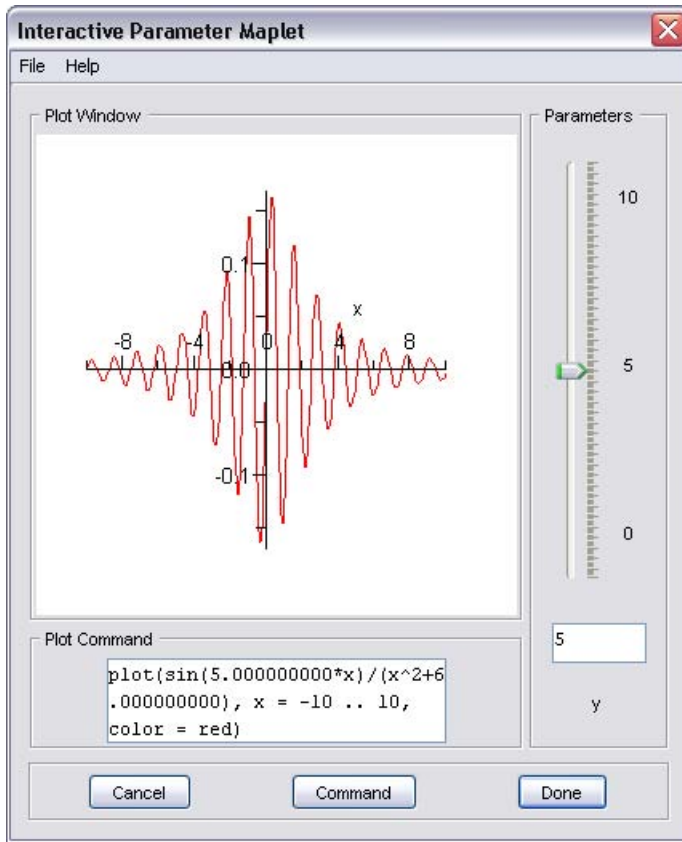
| Step                                                                | Details                                                                                                                                                                                                                                      |
|---------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Launch the <b>Interactive Plot Builder</b> and enter an expression. | Add the expression $1+4*\cos(4*\theta)$ .                                                                                                                                                                                                    |
| Change the x-axis range.                                            | In the <b>Select Plot Type</b> window: <ol style="list-style-type: none"> <li>a. Change the x axis range to <math>0 .. 8*\text{Pi}</math>.</li> </ol>                                                                                        |
| Set plot options.                                                   | In the <b>Plot Options</b> window: <ol style="list-style-type: none"> <li>a. From the <b>Coordinate System</b> group box, select <b>polar</b>.</li> <li>b. From the <b>Color</b> group box drop down menu, select <b>magenta</b>.</li> </ol> |
| Plot the expression.                                                | Click <b>Plot</b> .                                                                                                                                                                                                                          |



To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder* (page 208)

### Example 6 - Interactive Plotting

Using the **Interactive Plot Builder**, you can plot an expression with several of its variables set to numeric values. The **Interactive Parameter** window allows you to interactively adjust these numeric values within specified ranges to observe their effect.

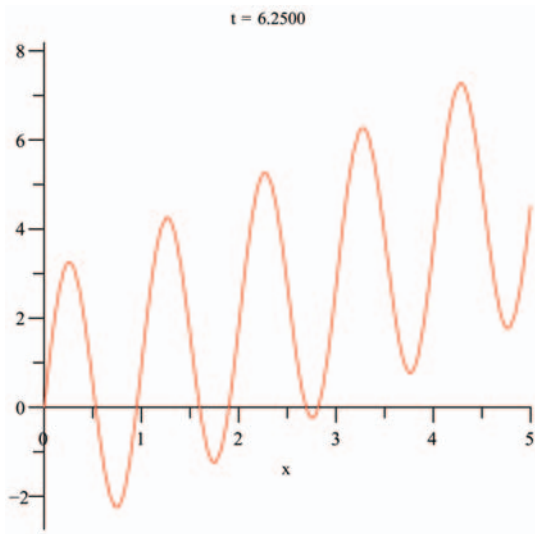


**Figure 5.1: Interactive Parameter Window**

**Table 5.7: Interactive Plotting**

| Steps                                                               | Details                              |
|---------------------------------------------------------------------|--------------------------------------|
| Launch the <b>Interactive Plot Builder</b> and enter an expression. | Add the expression $x+3*\sin(x*t)$ . |

| Steps               | Details                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|---------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Select a plot type. | <p>In the <b>Select Plot Type</b> window:</p> <ol style="list-style-type: none"><li>a. From the <b>Select Plot</b> group box, select<br/><b>Interactive Plot with 1 parameter</b>.</li><li>b. Change the range of the <b>x-axis</b> to <b>0 .. 5</b>.</li><li>c. Change the <b>t</b> range to <b>0 .. 10</b>.</li><li>d. Click <b>Plot</b> to launch the <b>Interactive Parameter</b> window.</li></ol> <p><b>Note:</b> To apply plot options before interactively adjusting the plot, click <b>Options</b> to launch the <b>Plot Options</b> window.</p> <p>After setting the plot options, click <b>Plot</b> to display the <b>Interactive Parameter window</b>.</p> |
| Adjust the plot.    | <ol style="list-style-type: none"><li>1. To adjust the numeric values, use the slider.</li><li>2. Click <b>Done</b> to return the plot to the Maple document.</li></ol>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |



To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder* (page 208)

For information on customizing plots using the **Interactive Plot Builder**, refer to Customizing Plots : *Interactive Plot Builder Options* (page 216).

## Context Menu

A context menu in Maple displays a list of commands to manipulate, display, or calculate using a Maple expression. The commands in the menu depend on the type of the expression. To display the context menu for a Maple expression, right-click (for Macintosh, **Control**-click) the expression.

For expressions, the context menu lists:

- 2-D or 3-D plot
- 2-D or 3-D implicit plot
- **Interactive Plot Builder**

based on the expression selected.

By invoking the **Interactive Plot Builder** through the context menu, the expression automatically passes to the builder and Maple does not display the **Specify Expression** window.

One advantage of using the context menu is the simplicity of creating an expression using menus. By using this method, you do not need any knowledge of plot command syntax.

1. Enter and evaluate an expression, for example,  $\frac{xy}{x^2+y^2}$ .
2. Right-click (**Control**-click for Macintosh) the expression.
3. From the context menu, select **Plots > 3-D Plot > x,y**.

>  $\frac{xy}{x^2 + y^2}$

$$\frac{xy}{x^2 + y^2}$$

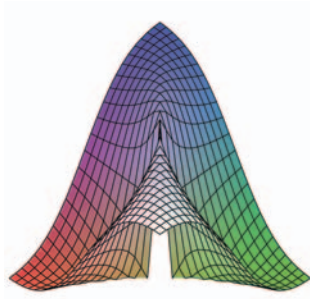
|                      |        |   |
|----------------------|--------|---|
| Cut                  | Ctrl+X |   |
| Copy                 | Ctrl+C |   |
| Paste                | Ctrl+V |   |
| Apply a Command      |        |   |
| Approximate          |        | ▶ |
| Assign to a Name     |        |   |
| Collect              |        | ▶ |
| Denominator          |        |   |
| Differentiate        |        | ▶ |
| Evaluate at a Point  |        |   |
| Expand               |        |   |
| Factor               |        |   |
| Integrate            |        | ▶ |
| Normal               |        |   |
| Numerator            |        |   |
| Simplify             |        |   |
| Solve for a Variable |        | ▶ |
| Complex Maps         |        | ▶ |
| Constructions        |        | ▶ |
| Conversions          |        | ▶ |
| Integer Functions    |        | ▶ |
| Integral Transforms  |        | ▶ |
| Language Conversions |        | ▶ |
| Plots                |        | ▶ |
| Simplifications      |        | ▶ |
| Sorts                |        | ▶ |
| Units                |        | ▶ |

|                   |   |
|-------------------|---|
| 2-D Implicit Plot | ▶ |
| 3-D Plot          | ▶ |
| Plot Builder      |   |

|      |
|------|
| x, y |
| y, x |



For information on customizing plots using the context menu, see *Context Menu Options* (page 217).

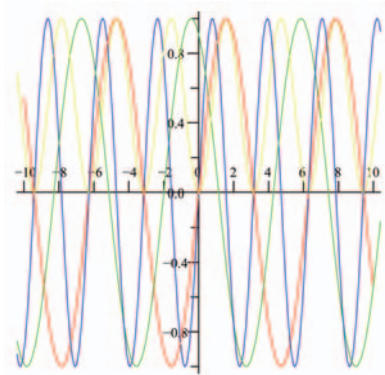
## Dragging to a Plot Region

To use the drag-and-drop method, use the plot region created by one of the other methods or insert an empty plot region into the document. Empty plot regions can be two-dimensional or three-dimensional.

Advantages of the drag-and-drop method include the ease of adding and removing plots and the independence from plotting command syntax.

1. From the **Insert** menu, select **Plot**, and then **2D**.
2. Enter the expression **sin(x)** in an input region.
3. Select the full expression in the input region and drag it into the plot region.
4. Repeat steps 2 and 3 using the following expressions: **sin(2\*x)**, **sin(x+2)**, and **sin(x)^2**.
5. To remove an expression from the plot region, drag-and-drop the expression plot from the plot region to a Maple input region.





## The plot and plot3d Commands

The final method for creating plots is entering plotting commands.

The main advantages of using plotting commands are the availability of all Maple plot structures and the greater control over the plot output. Plot options are discussed in *Customizing Plots* (page 216).

**Table 5.8: The plot and plot3d Commands**

**plot**(plotexpression, x=a..b, ...)

**plot3d**(plotexpression, x=a..b, y=a..b, ...)

- **plotexpression** - expression to be plotted
- **x=a..b** - name and horizontal range
- **y=a..b** - name and vertical range

### Maple commands from Creating Plots: Interactive Plot Builder

The following examples show the plotting commands returned by the examples in *Interactive Plot Builder* (page 191).

#### Example 1 - Display a plot of a single variable expression

```
> plot( sin(x)/x, x = -10..10 )
```

**Example 2 - Display a plot of multiple expressions of 1 variable**

To display multiple expressions in a plot, include the expressions in a list.

To enter  $\frac{d}{dx} \sin(x^2)$  and  $\int \sin(x^2) dx$ , use the **Expression** palette. For more information, see *Entering Expressions* (page 10).

```
> plot([sin(x^2), d/dx sin(x^2), int sin(x^2) dx], x=-3..3)
```

**Example 3 - Display a plot of a multi-variable expression**

```
> plot3d(1 + sin(x y) / (x^2 + y^2), x = -5..5, y = -5..5, view = 0..0.5
, lightmodel = light1, shading = zgrayscale
, style = patchnogrid, grid = [40, 40])
```

**Example 4 - Display a conformal plot**

A collection of specialized plotting routines are available in the **plots** package. For access to a single command in a package, use the long form of the command.

```
> plots[conformal](z^3, z = 0..2 + 2 I, axes = normal, grid = [20, 20])
```

**Example 5 - Display a plot in polar coordinates**

```
> plot(1 + 4 cos(4 theta), theta = 0..8 pi, coords = polar, color = magenta)
```

**Example 6 - Interactive Plotting**

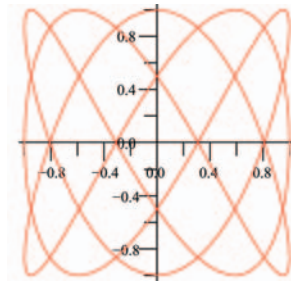
```
> plots[animate](plot, [x + 3 sin(x t), x = 0..5], t = 0..10)
```

For more information on the plot options described in this section, refer to the **?plot/options** and **?plot3d/options** help pages.

### Display a Parametric Plot

Some graphs cannot be specified explicitly. In other words, you cannot write the dependent variable as a function of the independent variable,  $y=f(x)$ . One solution is to make both the x-coordinate and the y-coordinate depend upon a parameter.

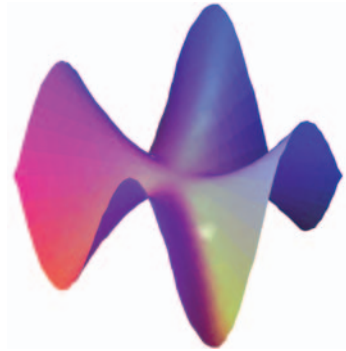
> `plot([cos(3 t), sin(5 t), t = 0..2 π])`



### Display a 3-D Plot

Maple can plot an expression of two variables as a surface in three-dimensional space. To customize the plot, include **plot3d** options in the calling sequence. For a list of plot options, see *The plot and plot3d Options (page 220)*.

```
> plot3d( $\frac{xy(x^2-y^2)}{x^2+y^2}$ , x=-2..2, y=-2..2, glossiness=0.5, style=
  patchnogrid, light=[100, 345, 50, 255, 255], ambientlight
  =[0.5, 0, 1])
```



## The plots Package

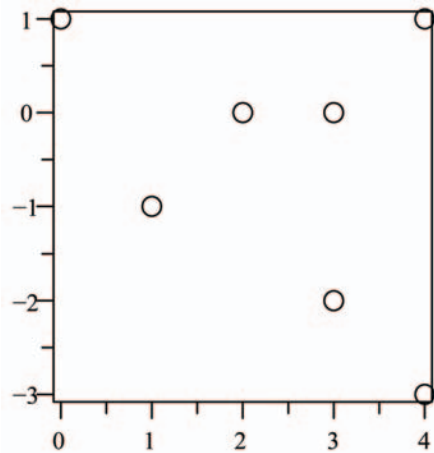
The **plots** package contains numerous plot commands for specialized plotting. This package includes: **animate**, **contourplot**, **densityplot**, **fieldplot**, **odeplot**, **matrixplot**, **spacecurve**, **textplot**, and **tubeplot**. For details about this package, refer to the **?plots** help page.

```
> with(plots):
```

### The pointplot Command

To plot numeric data, use the **pointplot** command in the **plots** package with the data organized in a list of lists structure of the form **[[x1, y1], [x2, y2], ..., [xn, yn]]**. By default, Maple does not connect the points. To draw a line through the points, use the **style = line** option. For further analysis of data points, use the **Curve Fitting Assistant**, (**Tools>Assistants>CurveFitting**) which fits and plots a curve through the points. For more information, refer to the **?CurveFitting[Interactive]** help page.

```
> pointplot([[0, 1], [1, -1], [3, 0], [4, -3], [2, 0], [4, 1], [3, -2], [4, 1]])  
      , axes = BOXED, symbolsize=25, symbol=circle)
```



### The `matrixplot` Command

The `matrixplot` command plots the values of a plot object of type **Matrix**. The `matrixplot` command accepts options such as **heights** and **gap** to control the appearance of the plot. For more information on Matrices, see *Linear Algebra* (page 135).

```
> with(LinearAlgebra):
```

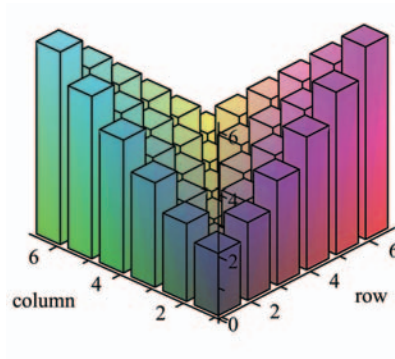
```
> A := HilbertMatrix(6)
```

$$A := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \end{bmatrix}$$

>  $B := \text{ToeplitzMatrix}([1, 2, 3, 4, 5, 6], \text{symmetric})$

$$B := \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 & 2 & 3 \\ 5 & 4 & 3 & 2 & 1 & 2 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

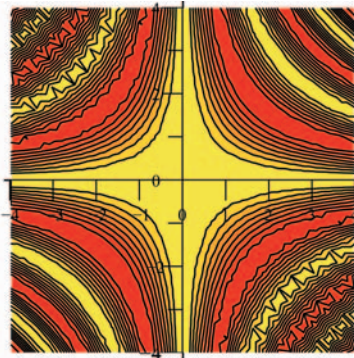
>  $\text{matrixplot}(A + B, \text{heights} = \text{histogram}, \text{axes} = \text{normal}, \text{gap} = 0.25, \text{style} = \text{patch})$



## The `contourplot` Command

The **`contourplot`** command generates a topographical map for an expression or function. To create a smoother, more precise plot, increase the number of points using the **`numpoints`** option.

```
> contourplot(cos(x*y), x = -4..4, y = -4..4, filled = true
, numpoints = 750)
```

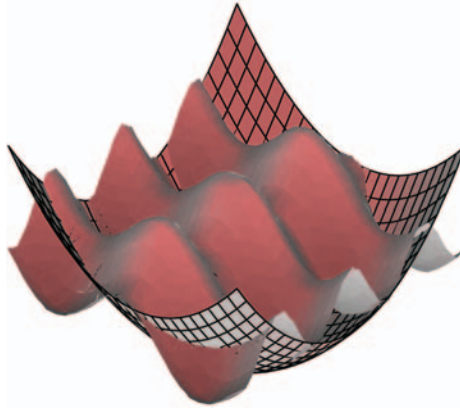


## Multiple Plots in the Same Plot Region

### List of Expressions

To display multiple expressions in the same plot region, enter the expressions in a **`list`** data structure. To distinguish the surfaces, apply different shading options, styles, or colors to each surface.

```
> plot3d([cos(5 x) + cos(5 y), x^2 + 3 y^2 - 4], x=-2..2, y=-1..1,
  shading=[zgrayscale, none], color=[default, grey], style
  =[patchnograd, patch], lightmodel=light3, transparency=0.1)
```



## The display Command

To display different types of plots in the same plot region, use the **display** command in the **plots** package.

This example plots a curve over a hill with the shadow of the curve projected onto the hill.

```
> with(plots):
```

```
> z := 10 ( x^2 + y^5 + x/5 ) e^(-x^2 - y^2);
```

```
> hill := plot3d(z, x = -2..2, y = -2.5..2.5, shading = zhue
  , style = patchnograd, lightmodel = light3
  , orientation = [-125, 60]):
```

```
> xt := cos(t):
```

```
> yt := 2 sin(t):
```



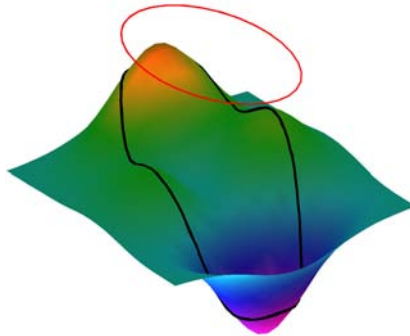
Maple can draw curves in three-dimensional space.

```
> curve := spacecurve([xt, yt, 10], t = 0..10, color = red, thickness = 2):
```

```
> zt := subs({x = xt, y = yt}, z):
```

```
> shadow :=  
    spacecurve([xt, yt, zt], t = - $\pi$ .. $\pi$ , color = black, thickness = 2):
```

```
> display(hill, curve, shadow)
```



Now that you have seen how easy it is to incorporate a plot into your work, the next section illustrates how to customize plots.

## 5.3 Customizing Plots

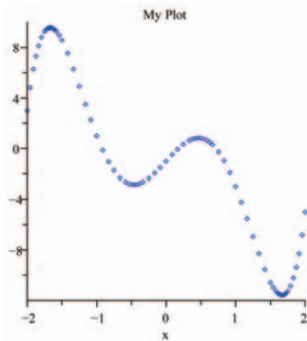
Maple provides many plot options to display the most aesthetically pleasing, illustrative results. Plot options include line styles, colors, shadings, axes styles, and titles where applicable. Plot options are applied using the **Interactive Plot Builder**, the context menus, or as options in the command syntax.

### Interactive Plot Builder Options

The **Interactive Plot Builder** offers most of the plot options available in Maple in an easy-to-use interface.

**Table 5.9: Customizing Plots Using Interactive Plot Builder**

| Steps                                                                | Details                                                                                                                                                                                                                                                                                                                                               |
|----------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Launch the <b>Interactive Plot Builder</b> and enter the expression. | Add the expression $2*x^5-10*x^3+6*x-1$ . For information on interacting with the <b>Interactive Plot Builder</b> , see <i>Example 1 - Display a plot of a single variable expression (page 193)</i>                                                                                                                                                  |
| Set the x-axis range.                                                | In the <b>Select Plot Type</b> window, change the x-axis range to <b>-2 .. 2</b> .                                                                                                                                                                                                                                                                    |
| Set plot options.                                                    | In the <b>Plot Options</b> window: <ol style="list-style-type: none"> <li>From the <b>Line</b> group box, select <b>dot</b>.</li> <li>From the <b>Color</b> group box, select <b>blue</b>.</li> <li>From the <b>Axes</b> group box, select <b>frame</b>.</li> <li>From the <b>Title</b> group box, enter <b>My Plot</b> in the text field.</li> </ol> |
| Plot the expression.                                                 | Click <b>Plot</b> .                                                                                                                                                                                                                                                                                                                                   |



## Context Menu Options

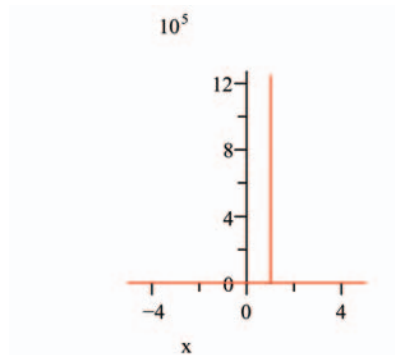
Using the context menu, you can alter a plot by right-clicking (for Macintosh, **Control**-clicking) the plot output. You can also access a large subset of plot

options using the **Plot** toolbar and **Plot** menu options. These menus display when a plot region is selected. Regardless of the method used to insert a plot into Maple, you can use the context menu to apply different plot options. For a list of options available when plotting in two and three dimensions, see *The plot and plot3d Options* (page 220).

## 2-D Plot Options

Some plots do not display as you would expect using default option values. A expression with a singularity is one such example.

>  $\text{plot}\left(\frac{1}{(x-1)^2}, x = -5..5\right)$



In the previous plot, all interesting details of the plot are lost because there is a singularity at  $x = 1$ . The solution is to view a narrower range, for example, from  $y = 0$  to  $7$ .

**Table 5.10: Customizing 2-D Plots Using the Context Menu**

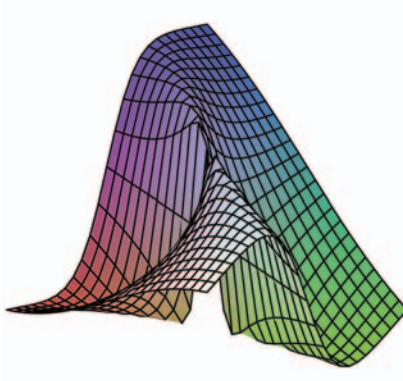
| Steps                   | Details                                                                                                                                                                                                                                                                                            |
|-------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Alter the y-axis range. | <ol style="list-style-type: none"> <li>1. Right-click the plot region. Select <b>Axes</b>, and then <b>Range</b>.</li> <li>2. In the <b>Axes Ranges</b> dialog, in the <b>y</b> group box, select the bottom radio button and enter <b>0</b> and <b>7</b> in the text regions provided.</li> </ol> |

| Steps                  | Details                                                                                                                                                                                                     |
|------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Change the color.      | Place the mouse pointer on the curve and right-click ( <b>Control</b> -click, for Macintosh). <b>Note:</b> The curve is selected when it becomes highlighted. Select <b>Color</b> , and then <b>Green</b> . |
| Change the line style. | Select <b>Style</b> , and then <b>Point</b> .                                                                                                                                                               |

### 3-D Plot Options

By default, Maple displays the graph as a shaded surface and scales the plot to fit the window. To change these options, use the context menu.

```
> plot3d(  $\frac{xy}{x^2 + y^2}$ , x = -10..10, y = -5..5 )
```



Maple has many preselected light source configurations.

**Table 5.11: Customizing 3-D Plots Using the Context Menu**

| Steps                 | Details                                                                                   |
|-----------------------|-------------------------------------------------------------------------------------------|
| Change the style.     | Right-click the plot region. Select <b>Style</b> , and then <b>Patch (Without Grid)</b> . |
| Apply a light scheme. | Select <b>Lighting</b> , and then <b>Light Scheme 1</b> .                                 |
| Change the color.     | Select <b>Color</b> , and then <b>Z (Grayscale)</b> .                                     |

| Steps                  | Details                                                                      |
|------------------------|------------------------------------------------------------------------------|
| Change the axes style. | Select <b>Axes</b> , and then <b>Boxed</b> .                                 |
| Alter the glossiness.  | Select <b>Glossiness</b> . Using the slider, adjust the level of glossiness. |

## The plot and plot3d Options

If you are using commands to insert a plot, you can specify plot options as arguments at the end of the calling sequence. You can specify the options in any order. Applying plot options in the command syntax offers a few more options and greater control than what is available in the **Interactive Plot Builder** and context menus.

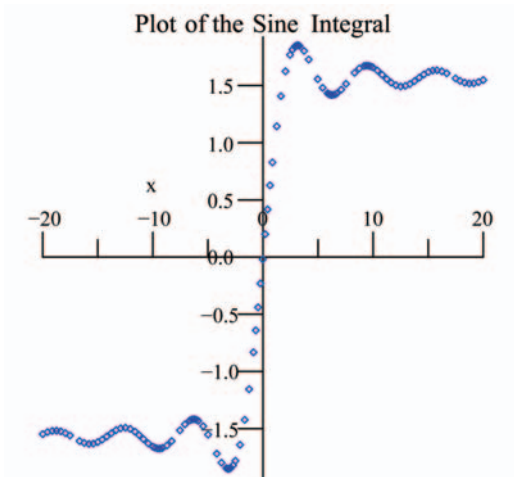
**Table 5.12: Popular Plot Options**

| Option           | Description                                                                                                                             |
|------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| axes             | Defines the type of axes, one of: <b>boxed</b> , <b>frame</b> , <b>none</b> , or <b>normal</b>                                          |
| color            | Defines a color for the curves to be plotted                                                                                            |
| font             | Defines the font for text objects in the plot                                                                                           |
| glossiness (3-D) | Controls the amount of light reflected from the surface                                                                                 |
| gridlines (2-D)  | Defines gridlines in the plot                                                                                                           |
| lightmodel (3-D) | Controls the light model to illuminate the plot, one of: <b>none</b> , <b>light1</b> , <b>light2</b> , <b>light3</b> , or <b>light4</b> |
| linestyle        | Defines the dash pattern used to render lines in the plot, one of: <b>dash</b> , <b>dashdot</b> , <b>dot</b> , or <b>solid</b>          |
| legend (2-D)     | Defines a legend for the plot                                                                                                           |
| numpoints        | Controls the minimum total number of points generated                                                                                   |
| scaling          | Controls the scaling of the graph, one of: <b>constrained</b> or <b>unconstrained</b>                                                   |
| shading (3-D)    | Defines how the surface is colored, one of: <b>xyz</b> , <b>xy</b> , <b>z</b> , <b>zgrayscale</b> , <b>zhue</b> , or <b>none</b>        |

| Option             | Description                                                                                                                                                                                                              |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| style              | Defines how the surface is to be drawn, one of: <b>line</b> , <b>patch</b> , <b>patchcontour</b> , <b>patchngrid</b> , or <b>point</b> for 2-D plots; <b>contour</b> , <b>hidden</b> , or <b>wireframe</b> for 3-D plots |
| symbol             | Defines the symbol for points in the plot, one of: <b>box</b> , <b>cross</b> , <b>circle</b> , <b>point</b> , or <b>diamond</b>                                                                                          |
| title              | Defines a title for the plot                                                                                                                                                                                             |
| thickness          | Defines the thickness of lines in the plot                                                                                                                                                                               |
| transparency (3-D) | Controls the transparency of the plot surface                                                                                                                                                                            |
| view               | Defines the minimum and maximum coordinate values of the curve displayed on the screen                                                                                                                                   |

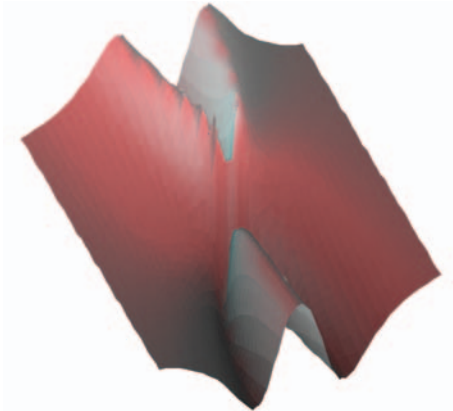
For a complete list of plot options, refer to the **?plot/options** and **?plot3d/options** help pages.

```
> plot(Si(x), x = -20..20, title = "Plot of the Sine Integral",
      , titlefont = [HELVETICA, 12], color = blue, style = point)
```



To create a smoother or more precise plot, calculate more points using the **numpoints** option.

```
> plot3d(
  (
     $\frac{xy^2}{x^2 + y^4}$ , x = -10..10, y = -10..10
    , axes=boxed, numpoints = 1500, lightmodel = light3
    , shading = zgrayscale, orientation = [160, 20]
    , style = patchnogrid
  )
)
```







## 5.4 Analyzing Plots

### Point Probe, Rotate, Pan, and Scale Tools

To gain further insight into a plot, Maple offers various tools to analyze plot regions. These tools are available in the **Plot menu** menu, **Context Bar** and in the context menu under **Transform** when the plot region is selected.

**Table 5.13: Plot Analysis Options**

| Name                 | Icon                                                                                | Description                                                                                                                        |
|----------------------|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| Point probe<br>(2-D) |  | Display the coordinates corresponding to the cursor position on a two-dimensional plot in the context bar (upper left-hand corner) |
| Rotate<br>(3-D)      |  | Rotate a three-dimensional plot to see it from a different point of view                                                           |

| Name  | Icon                                                                              | Description                                        |
|-------|-----------------------------------------------------------------------------------|----------------------------------------------------|
| Pan   |  | Change the position of the plot in the plot region |
| Scale |  | Change the size of the plot without resampling     |

## 5.5 Creating Animations

Plotting is an excellent way to represent information. Animations allow you to emphasize certain graphical behavior, such as the deformation of a bouncing ball, clearer than in a static plot. A Maple animation is a number of plot frames displayed in sequence, similar to the action of movie frames. To create an animation, use the **Interactive Plot Builder** or commands.

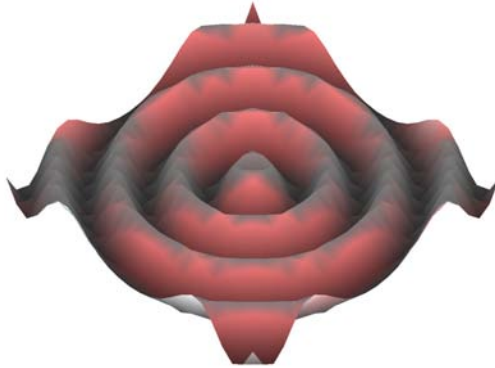
### Interactive Plot Builder

**Table 5.14: Creating Animations Using the Interactive Plot Builder**

| Steps                                                                | Details                                                                                                                                                                                                                                                                                                                                                                          |
|----------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Launch the <b>Interactive Plot Builder</b> and enter the expression. | Add the expression $\sin(i*\sqrt{x^2+y^2}/10)$ . For information on interacting with the <b>Interactive Plot Builder</b> , see <i>Example 1 - Display a plot of a single variable expression (page 193)</i> .                                                                                                                                                                    |
| Set axes and animation parameter range.                              | In the <b>Select Plot Type</b> window: <ol style="list-style-type: none"> <li>a. From the <b>Select Plot Type</b> drop-down menu, select <b>Animation</b>.</li> <li>b. Change the <b>x Axis</b> range to <b>-6 .. 6</b>.</li> <li>c. Change the <b>y Axis</b> range to <b>-6 .. 6</b>.</li> <li>d. Change the <b>Animation Parameter (i)</b> range to <b>1 .. 30</b>.</li> </ol> |

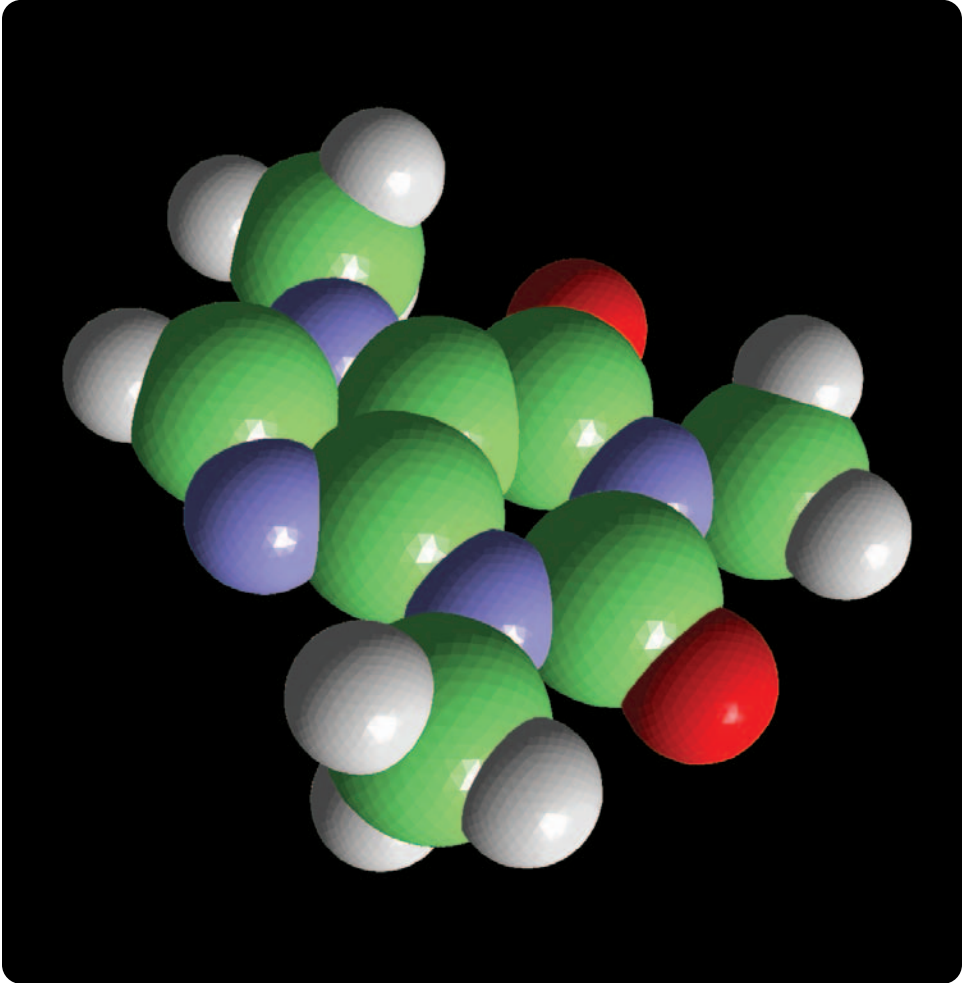


| Steps                | Details                                                                                                                                                                                                                                                                                                                                                                                                                               |
|----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Set plot options.    | In the <b>Plot Options</b> window:<br><br>a. From the <b>Style</b> group box, select <b>patch w/o grid</b> .<br><br>b. From the <b>Color</b> group box, in the <b>Light Model</b> drop-down menu select <b>red-turquoise</b> .<br><br>b. From the <b>Color</b> group box, in the <b>Shading</b> drop-down menu select <b>z (grayscale)</b> .<br><br>c. In the <b>View</b> group box, select the <b>Constrained Scaling</b> check box. |
| Plot the expression. | Click <b>Plot</b> .                                                                                                                                                                                                                                                                                                                                                                                                                   |

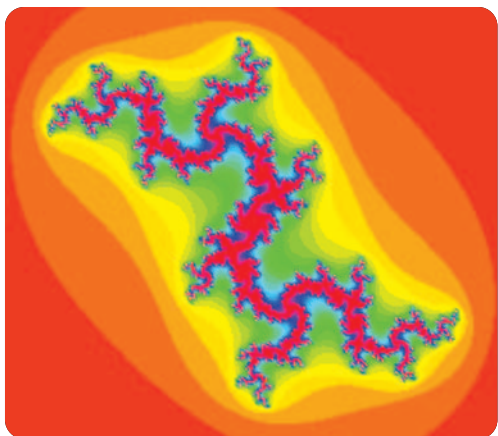


For information on playing the animation, see *Playing Animations* (page 226). To see the Maple syntax used to generate this plot, see *Maple Syntax for Creating Animations: Interactive Plot Builder Example* (page 225).

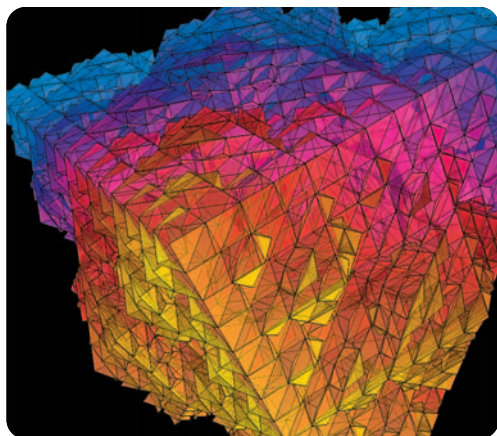
# Color Plates



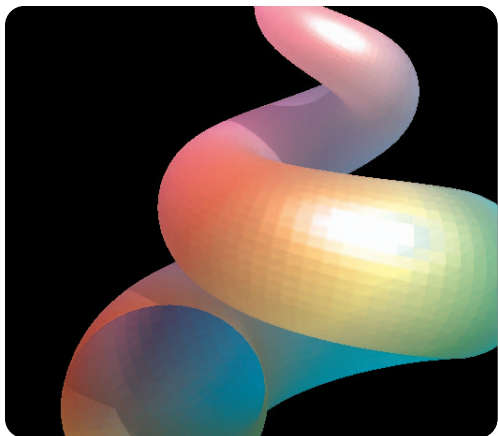
Caffeine Atom Model



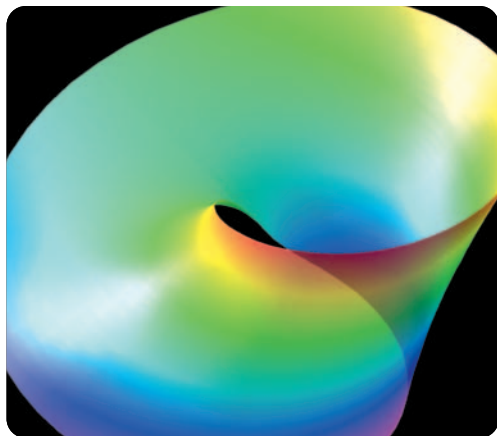
Julia Set



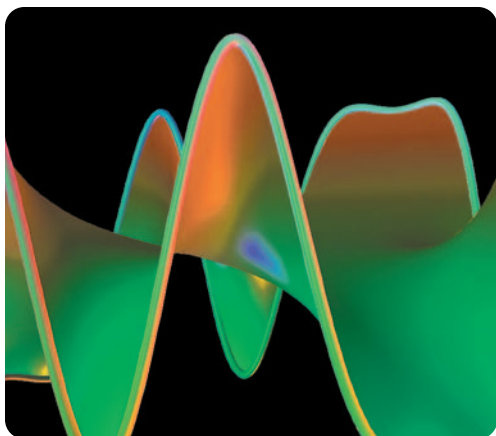
Koch Tetrahedron



Conchoid



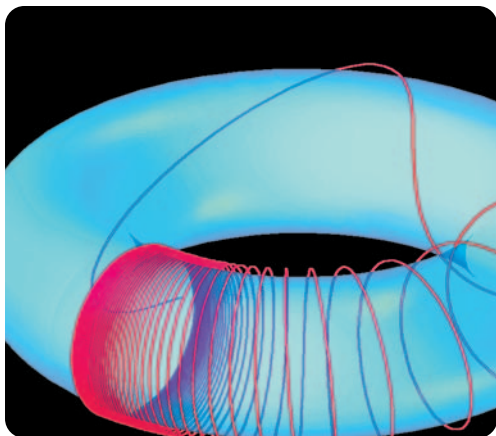
Möbius Strip



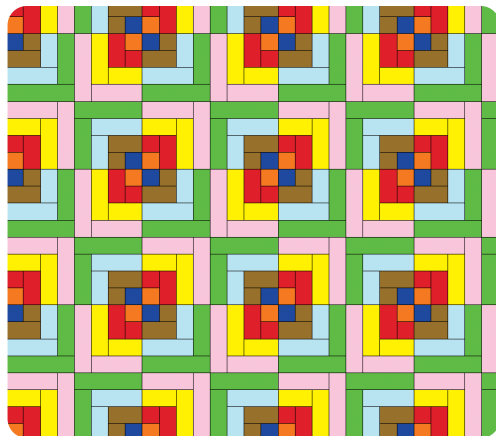
Dirichlet Problem for a Circle



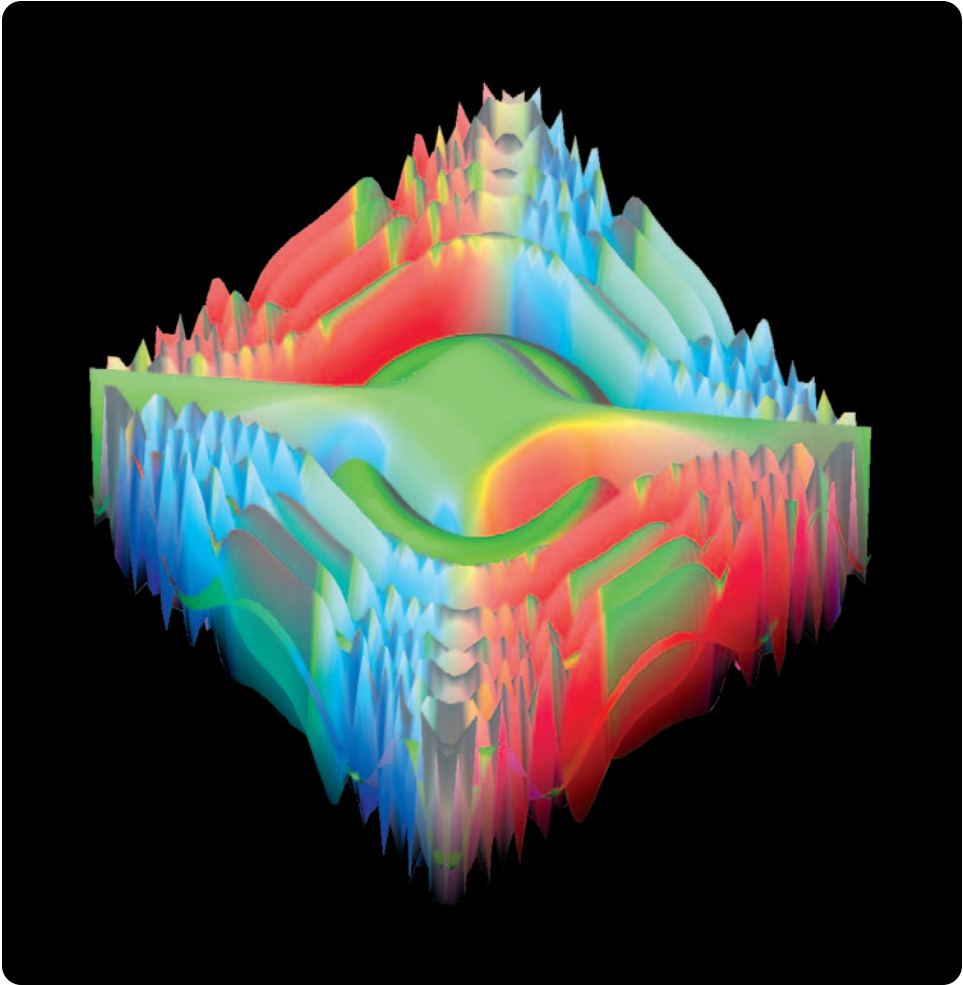
Fractal Leaf



Gauss Map Graphed on a Torus



Log Cabin Quilt



Function of Two Variables in Cartesian Coordinates

## The plots[animate] Command

You can also use the **animate** command, in the **plots** package, to generate animations.

**Table 5.15: The animate Command**

|                                                                                                                                                                                                                                                                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <pre>animate(plotcommand, plotarguments, t=a..b, ...)</pre>                                                                                                                                                                                                                                                                       |
| <pre>animate(plotcommand, plotarguments, t=L, ...)</pre>                                                                                                                                                                                                                                                                          |
| <ul style="list-style-type: none"> <li>• <b>plotcommand</b> - Maple procedure that generates a 2-D or 3-D plot</li> <li>• <b>plotarguments</b> - arguments to the plot command</li> <li>• <b>t=a..b</b> - name and range of the animation parameter</li> <li>• <b>t=L</b> - name and list of real or complex constants</li> </ul> |

To access the command, use the short form name after invoking the **with(plots)** command.

> *with(plots):*

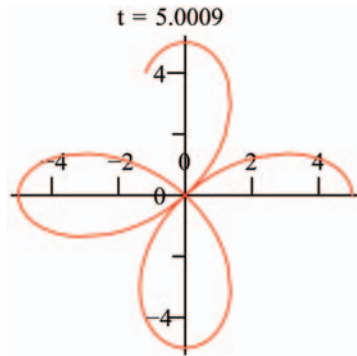
### Maple Syntax for Creating Animations: Interactive Plot Builder Example

The following example shows the plotting command returned by the example in *Interactive Plot Builder* (page 223).

```
> animate( plot3d, [ sin( (i*sqrt(x^2+y^2))/10 ), x = -6..6, y = -6..6
, style = patchnogrid, lightmodel = light3
, shading = zgrayscale, scaling = constrained ], i = 1..30 )
```

### Animate a 2-D plot

```
> animate(plot, [5 cos(2 θ), θ = 0..t, coords = polar], t = 0..2 π,
frames = 50)
```







For more information on the **animate** command, refer to the **?plots[animate]** help page.





## 5.6 Playing Animations

### Animation Context Bar

To run the animation, click the plot to display the **Animate** context bar.

**Table 5.16: Animation Options**

| Name           | Icon                                                                                | Description                               |
|----------------|-------------------------------------------------------------------------------------|-------------------------------------------|
| Previous Frame |  | View the previous frame in the animation. |
| Stop           |  | Stop the animation.                       |
| Play           |  | Play the selected animation.              |
| Next Frame     |  | View the next frame in the animation.     |

| Name                             | Icon                                                                                | Description                                                                                                                                                                                                                                       |
|----------------------------------|-------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Current Frame                    |    | <p>Slider control for viewing individual frames of an animated plot. The frame speed in frames per second (FPS) is displayed when increasing or decreasing the animation speed of a plot.</p>                                                     |
| Forward<br>Oscillate<br>Backward |    | <ul style="list-style-type: none"> <li>• Forward - Play the animation forward.</li> <li>• Oscillate - Play the animation forward and backward.</li> <li>• Backward - Play the animation backward.</li> </ul>                                      |
| Single<br>Continuous             |   | <ul style="list-style-type: none"> <li>• Single - Run the animation in single cycle mode. The animation is displayed only once.</li> <li>• Continuous - Run the animation in continuous mode. The animation repeats until you stop it.</li> </ul> |
| Frames per second                |  | <p>Set the animation to play at a faster or slower speed.</p>                                                                                                                                                                                     |

You can also run the animation using the context menu or the **Plot** menu.



## 5.7 Customizing Animations

The display options that are available for static plots are also available for Maple animations.

### Interactive Plot Builder Animation Options

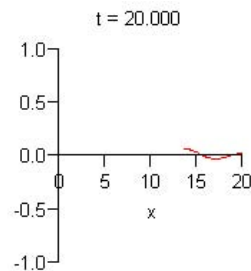
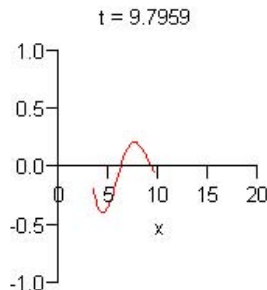
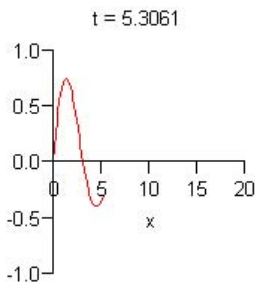
Using the **Interactive Plot Builder**, you can apply various plot options within the **Plot Options** window. See the *Interactive Plot Builder* (page 223) example.

### Context Menu Options

As with static plots, you can apply plot options to the animation by right-clicking (for Macintosh, **Control**-clicking) the animation output.

> *with(plots)* :

```
> animate(plot, [sin(x) e(-x/5), x=t-2π..t], t=0..20
           ,frames=50,view=[0..20, -1..1])
```



**Table 5.17: Customizing Animations Using the Context Menu**

| Step                  | Details                                                                    |
|-----------------------|----------------------------------------------------------------------------|
| Change the line style | Right-click the plot region. Select <b>Style</b> , and then <b>Point</b> . |
| Remove the axes       | Select <b>Axes</b> , and then <b>None</b> .                                |

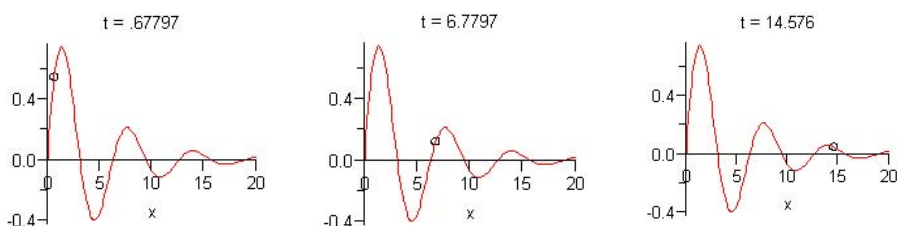
## The animate Command Options

The **animate** command offers a few options that are not available for static plots. Refer to the **?animate** help page for information on these additional options. By default, a two-dimensional animation consists of sixteen plots (frames) and a three-dimensional animation consists of eight plots (frames). To create a smoother animation, increase the number of frames using the **frames** option.

**Note:** Computing more frames increases time and memory requirements.

> *sinewave* := plot( $\sin(x) e^{-x/5}$ ,  $x = 0..20$ ):

> *animate*(*pointplot*  
 , [ [  $t, \sin(t) e^{-t/5}$  ], *symbol* = *circle*, *symbolsize* = 20 ]  
 ,  $t = 0..20$ , *frames* = 60, *background* = *sinewave*)



## 5.8 Exporting

You can export a generated graph or animation to an image in various file formats, including DXF, EPS, GIF, JPEG/JPG, POV, Windows BMP, and WMF. Exporting an animation to GIF produces an animated image file. The exported images can be included in presentations, Web pages, Microsoft Word, or other software.

### To export an image:

1. Right-click the plot region (for Macintosh, **Control**-click).
2. Select **Export** and the file format.

### Alternatively:

1. Click the plot.
2. From the **Plot** menu, select **Export**, and then the file format.

Maple has various plot drivers. By setting the **plotdevice**, a file can be automatically created without returning the image to the document. For more information, refer to the **?plotdevice** help page.

## 5.9 Code for Color Plates

Generating impressive graphics in Maple can require only a few lines of code, as shown by the examples in this chapter. However, other graphics require many lines of code. Code for the color plates is available at the Maple Application Center.

From the **Help** menu, select **Manuals, Dictionary, and more, On the Web**, and then **Application Center**.

To access the color plate code:

1. Go to the Maple Application Center.
2. Search for Color Plates.

# 6 Creating Mathematical Documents

Maple allows you to create powerful documents as business and education tools, technical reports, presentations, assignments, and handouts.

You can:

- Place instructions and equations side by side
- Format text for reports or course material
- Insert hyperlinks to other Maple files, Web sites, or email addresses
- Insert images, tables, and symbols
- Generate two- and three-dimensional plots and animations
- Sketch in the document
- Copy, cut, and paste information
- Bookmark specific areas
- Easily update, revise, and distribute your documents

This User Manual was written using Maple.

## 6.1 In This Chapter

| Section                                                                             | Topics                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Document Formatting - Add various formatting elements                               | <ul style="list-style-type: none"><li>• Quick Character Formatting</li><li>• Quick Paragraph Formatting</li><li>• Copy and Paste</li><li>• Sections</li><li>• Displaying Hidden Formatting Attributes</li><li>• Indentation and the Tab Key</li><li>• Character and Paragraph Styles</li><li>• Document Blocks</li><li>• Typesetting</li><li>• Using Tables for Layout</li><li>• Formatting Lists: Bullets, Numbers, Indent</li><li>• Bookmarks</li><li>• Inserting Images</li><li>• Show or Hide Document Content</li></ul> |
| Embedded Components - Insert buttons, sliders, and more in your document            | <ul style="list-style-type: none"><li>• Adding Graphical Interface Components</li><li>• Editing Component Properties</li><li>• Removing Graphical Interface Components</li><li>• Example Component Properties</li><li>• Printing and Exporting with Embedded Components</li></ul>                                                                                                                                                                                                                                            |
| Creating Graded Assignments - Create documents for automated testing and assessment | <ul style="list-style-type: none"><li>• Creating a Question</li><li>• Viewing Questions in Maple</li><li>• Saving Test Content</li></ul>                                                                                                                                                                                                                                                                                                                                                                                     |

| Section                                                                                                          | Topics                                                                                                                                                                                                        |
|------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Auto-execute - Execute selected regions of your document                                                         | <ul style="list-style-type: none"> <li>• Setting the Auto-Execute Feature</li> <li>• Removing the Auto-Execute Setting</li> <li>• Repeating Auto-Execution</li> <li>• Security Levels</li> </ul>              |
| Sketch Regions - Sketch an idea in the document by inserting a sketch pad                                        | <ul style="list-style-type: none"> <li>• Insert a Sketch Pad</li> <li>• Drawing</li> <li>• Canvas Style of Sketch Pad</li> <li>• Erase or Clear Content</li> <li>• Selection Tool</li> </ul>                  |
| Spell Checking - Verify text with the Maple spell checking utility                                               | <ul style="list-style-type: none"> <li>• How to Use the <b>Spellcheck</b> Utility</li> <li>• Selecting a Suggestion</li> <li>• <b>Spellcheck</b> Usage and the Document</li> <li>• User Dictionary</li> </ul> |
| Hyperlinks - Add hyperlinks to various sources                                                                   | <ul style="list-style-type: none"> <li>• Inserting a Hyperlink in the Document</li> <li>• Linking to an Email Address, Dictionary Topic, Help Page, Maplet Application, Web Page, or Document</li> </ul>      |
| Worksheet Compatibility - Classic Worksheet interface does not support all Standard Worksheet interface features | <ul style="list-style-type: none"> <li>• Compatibility Issues</li> </ul>                                                                                                                                      |

## 6.2 Document Formatting



### Quick Character Formatting

The **Format>Character** menu provides access to the following quick formatting features: **Bold**, **Italic**, **Underline**, **Superscript**, **Subscript**, font **Color**, and **Highlight Color**.

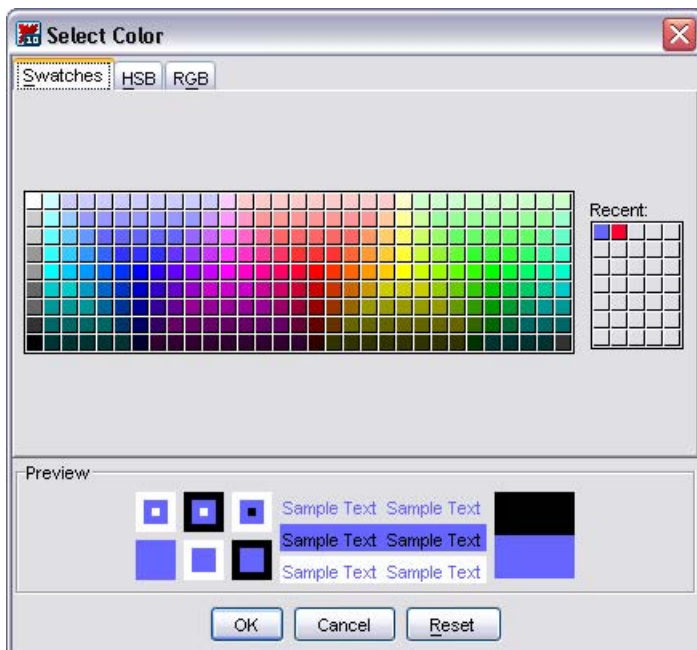
**To modify text:**

1. In the document, select the text to modify.
2. From the **Format** menu, select **Character**, and then the appropriate feature.

Alternatively, use the context bar icons.

- Font Color Context Bar Icon 
- Highlight Color Context Icon 

For font and highlight colors, you can select from Swatches, HSB, or RGB values. See Figure 6.1.



**Figure 6.1:** Select Color Dialog

## Attributes Submenu: Setting Fonts, Character Size, and Attributes

You can change various character attributes such as font, character size, style, and color in one dialog.

### To modify text:

1. In the document, select text to modify.
2. From the **Format** menu, select **Character**, and then **Attributes**. The **Character Style** dialog opens. See Figure 6.2.

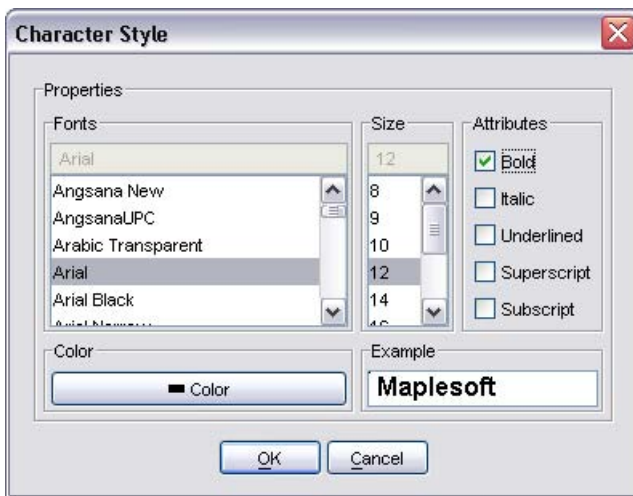


Figure 6.2: Character Style Dialog

## Quick Paragraph Formatting

The **Format>Paragraph** menu provides access to the following quick alignment features: **Align Left**, **Center**, **Align Right**, and **Justify**.

### To modify a paragraph:

1. In the document, select the paragraph to modify.
2. From the **Format** menu, select **Paragraph**, and then the appropriate feature.



## Attributes Submenu: Spacing, Indent, Alignment, Bullets, Line Break, and Page Break

You can change various paragraph attributes in one dialog.

- From the **Format** menu, select **Paragraph**, and then **Attributes**. The **Paragraph Style** dialog opens. See Figure 6.3.
- When changing spacing, you must indicate units (inches, centimeters, or points) in the **Units** drop-down list.

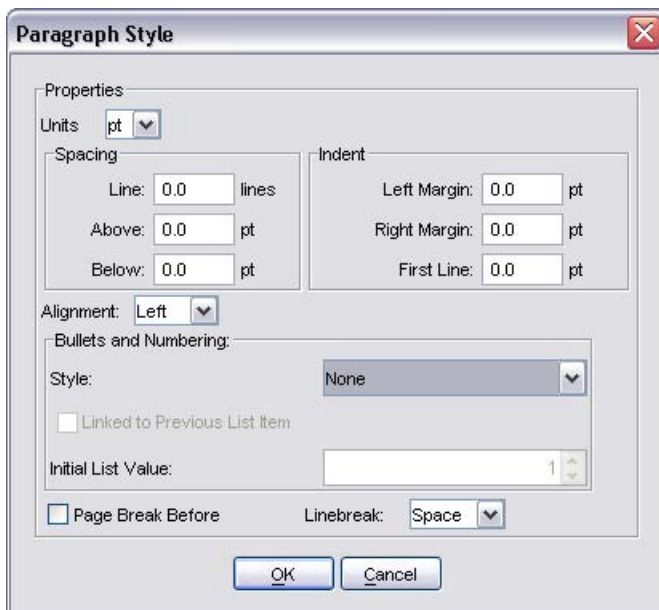


Figure 6.3: Paragraph Style Dialog

## Copy and Paste

You can cut, copy, and paste content in Maple documents.

**To copy an expression, or part of an expression, to another location on the document:**

1. Select the expression, or part of the expression, to copy.

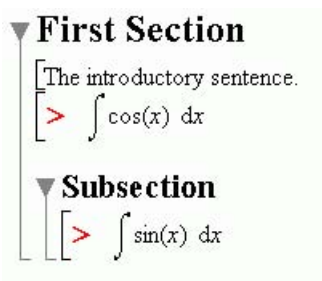
2. From the **Edit** menu, select **Copy**.
3. Place the cursor at the insertion point.
4. From the **Edit** menu, select **Paste**.

If you paste into an input region, Maple interprets all the pasted content as input. If you paste into a text region, Maple interprets all the pasted content as text. Note, however, that 2-D Math retains its format in both input and text regions.

When you copy and paste to another application, in general, Maple retains the original structure.

## Sections

You can organize your document into sections.





### Using the Insert Menu to Add Sections

1. Place the cursor in the paragraph or execution group above the location at which to insert a new section.
  - If the cursor is inside a section, Maple inserts the new section after the current section.
  - If the cursor is in an execution group, Maple inserts the new section after the execution group.
2. From the **Insert** menu, select **Section**. An arrow marks the start of the section.

3. Enter the section heading.
4. Press the **Enter** key.
5. Enter the body of the section.

### Using the Indent and Outdent Toolbar Icons

You can shift sections to create or remove subsections.

|                                                                                   |                                       |
|-----------------------------------------------------------------------------------|---------------------------------------|
|  | Enclose the selection in a subsection |
|  | Outdent the selection                 |

### Display Hidden Formatting Attributes


You can display icons that indicate the presence of hidden formatting attributes in the document, such as document block boundaries, execution groups marked for autoexecute, and bookmarks.



**To activate the marker feature:**

- From the **View** menu, select **Markers**. A vertical bar is displayed along the left pane of the document. Icons for hidden elements are displayed in the vertical bar next to the associated content in the document.

### Indentation and the Tab Key

The Tab icon allows you to set the **Tab** key to move between placeholders or to indent.

|                                                                                     |                                                                                                                                                                                                                                                                                                     |
|-------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | Tab icon <b>off</b> . Allows you to move between placeholders using the <b>Tab</b> key. As an illustration, click the exponent button in the <b>Expression</b> palette. The expression is inserted with the first placeholder highlighted. To move to the next placeholder, use the <b>Tab</b> key. |
|-------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|                                                                                        |                                                                                                                                            |
|----------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Text  | The Tab icon is disabled when using 2-D Math ( <b>Math</b> mode), and as such, the <b>Tab</b> key allows you to move between placeholders. |
|       | Tab icon <b>on</b> . Allows you to indent in the document using the <b>Tab</b> key.                                                        |

## Character and Paragraph Styles

Maple has predefined styles for characters and paragraphs. A style is a set of formatting characteristics that you can apply to text in your document to change the appearance of that text. When you apply a style, you apply a group of formats in one simple action.

- A **paragraph style** controls all aspects of a paragraph's appearance, such as text alignment, line spacing, and indentation. In Maple, each paragraph style includes a character style.
- A **character style** controls text font, size, and color, and attributes, such as bold and italic. To override the character style within a paragraph style, you must apply a character style or character formatting.

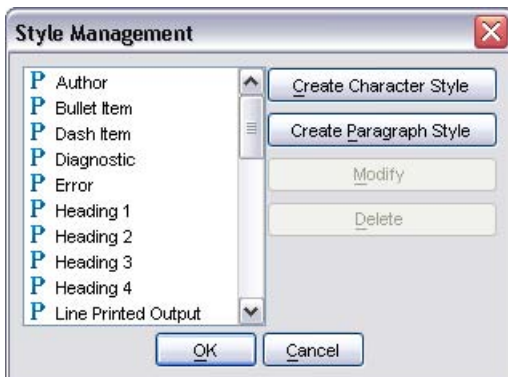


Figure 6.4: Style Management Dialog

### Applying Character Styles

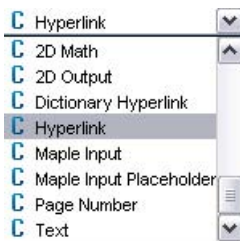
By using the drop-down list in the document context bar, you can apply:

- Existing Maple character styles.

- New styles that you have created through the **Style Management** (Figure 6.4) and **Character Style** (Figure 6.5) dialogs.

### To apply a character style to text in your document:

1. Select the text to modify.
2. In the styles drop-down list in the context bar of your document, select an appropriate character style. All character styles are preceded by the letter **C**. The selected text now reflects the attributes of the character style you have chosen.



3. (Optional) If necessary, you can remove this style. From the **Edit** menu, select **Undo**.

### Creating Character Styles

You can create custom character styles to apply to text. New styles are listed in the styles drop-down list in the context bar of your document.

1. From the **Format** menu, select **Styles**. The **Style Management** dialog opens. See Figure 6.4.
2. Click **Create Character Style**. The **Character Style** dialog opens. See Figure 6.5.
3. In the **Style** group box, enter a style name in the blank text field.
4. Select the properties for the new character style, such as **Font**, **Size**, **Attributes**, and **Color**. In the **Attributes** group box, the **Superscript** and **Subscript** check boxes are mutually exclusive. When you select one of the

two check boxes, the other is disabled. You must clear one before selecting the other.

**Note:** A preview of the style is displayed in the **Example** group box at the bottom of the **Character Style** dialog.

5. To create the style, click **OK** or to abandon creation, click **Cancel**.



**Figure 6.5: Character Style Dialog**

## Modifying Character Styles

### To modify character styles:

1. From the **Format** menu, select **Styles**. The **Style Management** dialog opens. See Figure 6.4.
2. From the style list, select the style to modify.
3. Click **Modify**. The **Character Style** dialog opens with the current attributes displayed. See Figure 6.5.

4. Select the properties to modify, such as **Font**, **Size**, **Attributes**, and **Color**. In the **Attributes** group box, the **Superscript** and **Subscript** check boxes are mutually exclusive. When you select one of the two check boxes, the other is disabled. You must clear one before selecting the other.

A preview of the style is displayed in the **Example** group box at the bottom of the **Character Style** dialog.

5. To accept changes, click **OK** or to cancel changes, click **Cancel**.

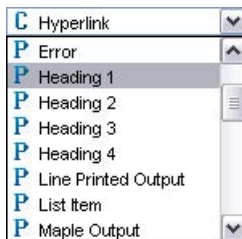
## Applying Paragraph Styles

By using the drop-down list in the document context bar, you can apply:

- Existing Maple paragraph styles.
- New styles that you have created through the **Style Management** (Figure 6.4) and **Paragraph Style** (Figure 6.6) dialogs.

### To apply a Maple paragraph style to text in your document:

1. Select the text to modify.
2. In the styles drop-down list in the context bar of your document, select an appropriate paragraph style. All Maple paragraph styles are preceded by the letter **P**. The selected text now reflects the attributes of the paragraph style you have chosen.



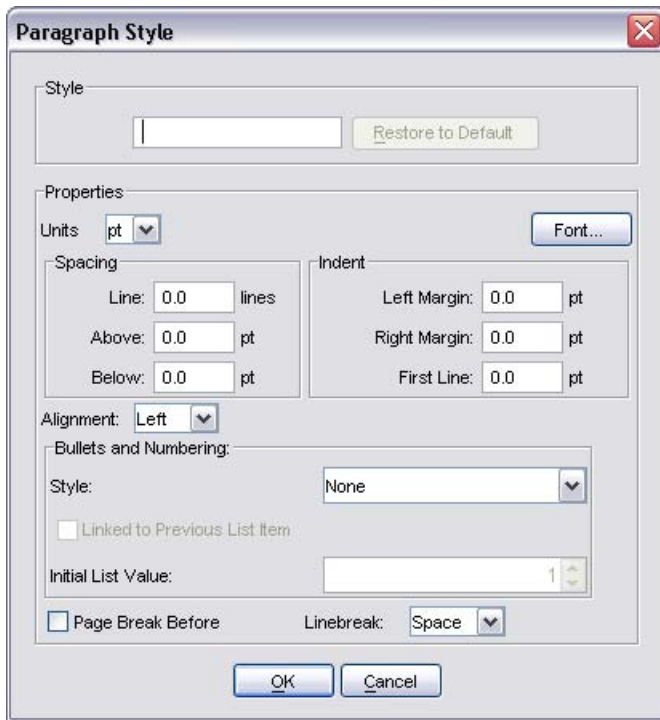
3. (Optional) If necessary, you can remove this style. From the **Edit** menu, select **Undo**.

## Creating Paragraph Styles

You can create custom paragraph styles to apply to text. New styles are listed in the styles drop-down list in the context bar of your document.

1. From the **Format** menu, select **Styles**. The **Style Management** dialog opens. See Figure 6.4.
2. Click **Create Paragraph Style**. The **Paragraph Style** dialog opens. See Figure 6.6.
3. In the **Style** group box, enter the new paragraph style name in the blank text field.
4. In the **Units** drop-down list, select the units used to determine spacing and indentation. Select from inches (**in**), centimeters (**cm**), or points (**pt**).
5. Select the properties to use for this paragraph style, such as **Spacing**, **Indent**, **Justification**, **Bullet Style**, **Linebreak**, and **Page Break Before**.
6. To add a font style, click **Font**. The **Character Style** dialog opens. For detailed instructions, see *Creating Character Styles* (page 240).
7. To create the style, click **OK**, or to abandon creation, click **Cancel**.





**Figure 6.6: Paragraph Style Dialog**

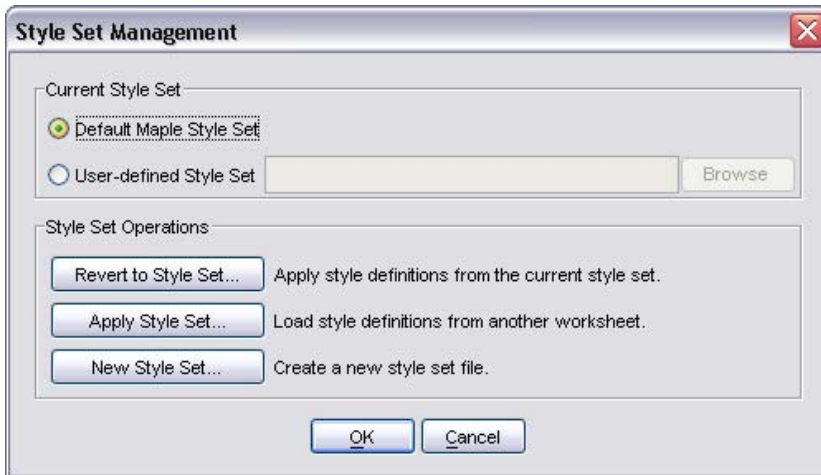
## Modifying Paragraph Styles

### To modify a paragraph style:

1. From the **Format** menu, select **Styles**. The **Style Management** dialog opens. See Figure 6.4.
2. Select a paragraph style to modify, and click **Modify**. The **Paragraph Style** dialog opens with the current attributes displayed.
3. Select the properties you want to modify, such as **Spacing**, **Indent**, **Justification**, and **Bullet**, **Linebreak**, and **Units**.
4. To modify the existing font style, click **Font**.
5. To accept changes, click **OK**, or to cancel changes, click **Cancel**.

## Style Set Management: Saving Styles for Future Use

You can use the style set of a particular document as the default style for all documents.



**Figure 6.7: Style Set Management Dialog**

## Creating and Applying Style Sets

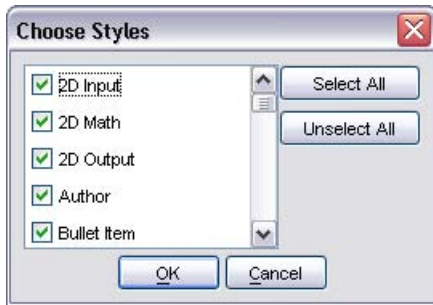
- Task 1 - Create Styles
- Task 2 - Create a New Style Set
- Task 3 - Apply a (New) Style Set

### TASK 1 - Create Styles:

- Create paragraph or character styles for the current document.

### TASK 2 - Create a New Style Set:

1. From the **Format** menu, select **Manage Style Sets**. The **Style Set Management** dialog opens. See Figure 6.7.
2. In the **Style Set Operations** group box, click **New Style Set**. The **Choose Styles** dialog opens. See Figure 6.8.



**Figure 6.8: Choose Styles Dialog**

3. Select all the styles that are part of your document style set. For example, if you modified the **Author** paragraph style to justify left versus the default style of centered, ensure that you have selected the **Author** check box in the **Choose Styles** dialog.
4. Click **OK**. The **Choose Filename** dialog opens.
5. Save your style set. The style is now available for future use in other documents.

### **TASK 3 - Apply a (New) Style Set:**

1. From the **Format** menu, select **Manage Style Sets**. The **Style Set Management** dialog opens. See Figure 6.7.
2. In the **Style Set Operations** group box, click **Apply Style Set**. The **Choose Filename** dialog opens.
3. Select the style file and click **Open**. The **Choose Styles** dialog opens. At this point, you can overwrite all the styles in your current document with the new style set or apply only a few.
4. Click **OK**. The style set is applied to your document.

### **Reverting to a Style Set**

At any point, you can revert your document style set to the **Default Maple Style Set** or to a **User-defined Style Set**.

**To revert to a style set:**

1. From the **Format** menu, select **Manage Style Sets**. The **Style Set Management** dialog opens.
2. In the **Current Style Set** group box, select the **Default Maple Style Set** or **User-defined Style Set**. For user-defined style sets, navigate (click **Browse**) to the file (**Choose Filename** dialog) and open the file (click **Open**).
3. In the **Style Set Operations** group box, click **Revert to StyleSet**.
4. In the **Choose Styles** dialog, select all the styles to revert, that is, overwrite with either the **Default Maple Style Set** or the **User-defined Style Set**.
5. Click **OK**.

## Document Blocks

With document blocks, you can create documents that present text and math in formats similar to those found in business and education documents.

In a document block, note the following.

- An input prompt or execution group is not displayed
- Warning messages are not displayed

By hiding Maple input such that only text and results are visible, you create a document with better presentation flow. Before using document blocks, it is recommended that you display **Markers**. A vertical bar is displayed along the left pane of the document. Icons representing document blocks are displayed in this vertical bar next to associated content.

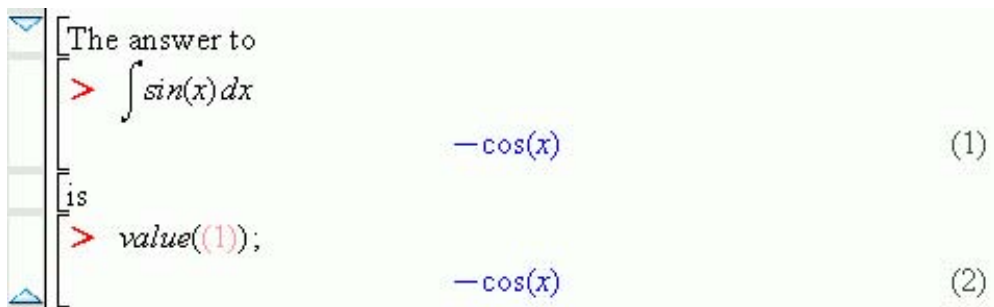
**To activate Markers:**

- From the **View** menu, select **Markers**.

## Applying Document Blocks: General Process

### To apply a document block to selected content:

1. Enter input at the Maple command prompt, creating input that can be referenced elsewhere in the document. See the **?EquationLabels** help page.
2. Execute the area, creating output that can be referenced elsewhere in the document.
3. Intersperse the area with content that is to remain visible, adding references to the input and output in the appropriate locations.



The answer to

$\int \sin(x) dx$

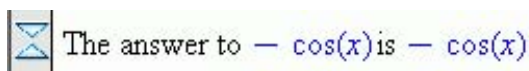
$-\cos(x)$  (1)

is

$\text{value}((1));$

$-\cos(x)$  (2)

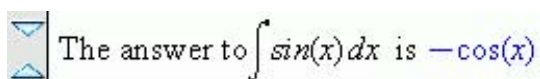
4. Select the entire area (text and math content) to format.
5. From the **Format** menu, select **Create Document Block**.



The answer to  $\int \sin(x) dx$  is  $-\cos(x)$

The block displays text and output only. You can select areas to display input only.

6. Select the output region you want to display as input. From the **View** menu, select **Toggle Input/Output Display**. The selected region displays input.



The answer to  $\int \sin(x) dx$  is  $\int \sin(x) dx$   $-\cos(x)$

## Working in Document Mode

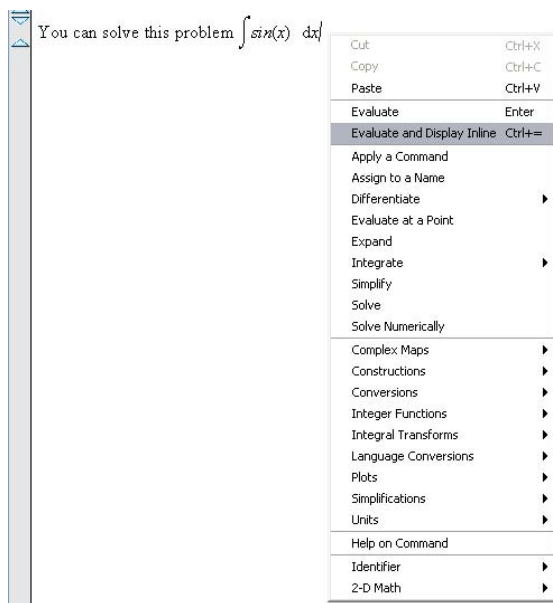
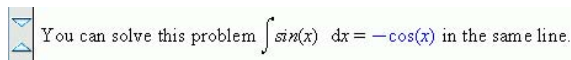
You can work directly in a document block in Worksheet mode, or you can work directly in Document mode, entering text and expressions, and then evaluating expressions.

### To start a document in Document mode:

1. From the **File** menu, select **New**, and then **Document Mode**. A document opens with the Document mode markers indicated in the left margin. Note that margin markers are visible if you select **View>Markers**.
2. Enter text and an expression to evaluate.
3. Select the expression and right-click (**Control**-click, for Macintosh) to display the context menu.
4. Click the **Evaluate and Display Inline** menu item. The expression is evaluated.

**Note:** Each time you press **Enter**, a new document block appears. Documents consist of a series of document blocks.

In the following figures, note how the expression is entered as part of the text and then evaluated with the context menu option **Evaluate and Display Inline**.

**Before****After****Figure 6.9: Working in Document Mode**

## View Document Code

To view the contents, that is, all code and expanded execution groups within a document block, you must expand the document block.

1. Place the cursor in the document block region.
2. From the **View** menu, select **Expand Document Block**.
3. To hide code again, select **View>Collapse Document Block**.

## Expand an Execution Group within a Document Block

An execution group is a grouping of Maple input with its corresponding Maple output. It is distinguished by a large square bracket at the left called a group boundary.

As document blocks can contain many execution groups, you can select to expand an execution group within a document block.

1. Place the cursor in the document block region.
2. From the **View** menu, select **Expand Execution Group**.
3. To hide the group, select **View>Collapse Execution Group**.

### **Switch between Input and Output**

1. Place the cursor in the document block region.
2. From the **View** menu, select **Toggle Input-Output Display**.

Input is displayed in one instance, or only output is displayed.

### **Inline Document**

Document blocks can display content inline, that is, text, input, and output in one line as presented in business and education documents.

#### **To display content inline:**

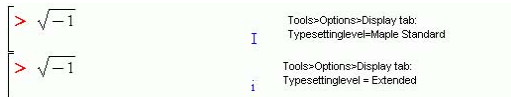
1. Place the cursor in the document block.
2. From the **View** menu, select **Inline Document Output**.



## Typesetting

You can control typesetting and 2-D Math equation parsing options in the Standard Worksheet interface. Extended typesetting uses a customizable set of rules for displaying expressions.

The rule-based typesetting functionality is available when **Typesettinglevel** is set to **Extended** (**Tools>Options>Display** tab). This parsing functionality applies to 2-D Math editing (**Math** mode) only.



To specify rules, use the **Typesetting Rule Assistant**.

- From the **View** menu, select **Typesetting Rules**. The **Typesetting Rule Assistant** dialog opens.

For more information, see the **?Typesetting**, **?TypesettingRuleAssist**, and **?OptionsDialog** help pages.

## Using Tables for Layout

Tables allow you to organize content in a document.

### Creating a Table

To create a table:

1. From the **Insert** menu, select **Table**.
2. Specify the number of rows and columns in the table creation dialog.
3. Click **OK**.

The default properties for the table include visible borders and auto-adjustment to 100% of the document width. These options, as well as the table

dimensions, can be modified after table creation. The following is an example table using the default settings.

|  |  |
|--|--|
|  |  |
|  |  |



## Cell Contents

Any content that can be placed into a document can also be placed into a table cell, including other sections and tables. Table cells can contain a mix of:

- Input commands
- 2-D Math
- Embedded components - buttons, sliders, check boxes, and more
- Plots
- Images

## Navigating Table Cells

Use the **Tab** key to move to the next cell.

|                                                                                     |                                                                                  |
|-------------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
|   | Tab icon <b>off</b> . Allows you to move between cells using the <b>Tab</b> key. |
|  | Tab icon <b>on</b> . Allows you to indent in the table using the <b>Tab</b> key. |

## Modifying the Structural Layout of a Table

The number of rows and columns in a table are modified using the **Insert** and **Delete** submenus in the **Table** menu or by using the **Cut** and **Paste** tools.

## Inserting Rows and Columns

Row and column insertion is relative to the table cell that currently contains the cursor. If the document has an active selection, insertion is relative to the selection boundaries.

- Column insertion can be to the left or right of the document position marker or selection.
- Row insertion can be above or below the marker or selection.

## Deleting Rows and Columns

With deleting operations using the **Delete** key, the **Delete Table Contents** dialog opens allowing you to specify the desired behavior. For example, you can delete the selected rows, or delete the contents of the selected cells.

## Pasting

Pasting a table subselection into a table may result in the creation of additional rows or columns, overwriting existing cell content, or the insertion of a subtable within the active table cell.

## Merging

You can merge cells across row or column borders. See Figure 6.10. The resultant cell must be rectangular. The contents of the individual cells in the merge operation are concatenated in execution order. See Figure 6.11.



Figure 6.10 shows two adjacent table cells. The left cell contains the text `> a + b;` on the top line and `a + b` on the bottom line. The right cell contains the text `> c + d;` on the top line and `c + d` on the bottom line.

Figure 6.10: Two cells

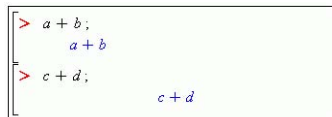


Figure 6.11 shows a single table cell that is the result of merging the two cells from Figure 6.10. The text is concatenated in execution order: `> a + b;` on the top line, `a + b` on the second line, `> c + d;` on the third line, and `c + d` on the bottom line.

Figure 6.11: Merged Cells

## Modifying the Physical Dimensions of a Table

The overall width of the table can be controlled in several ways.

The most direct way is to press the left mouse button (press mouse button, for Macintosh) while hovering over the left or right table boundary and dragging the mouse left or right. Upon release of the mouse button, the table boundary is updated. This approach can also be used to resize the relative width of table columns.

Alternatively, the size of the table can be controlled from the **Table Properties** dialog. Select **Tables>Properties**. Two sizing modes are supported.

(1) **Fixed percentage of page width**. Using this option, the table width adjusts whenever the width of the document changes. This option is useful for ensuring that the entire content of the table fits in the screen or printed page.

(2) **Scale with zoom factor**. This option is used to preserve the size and layout of the table regardless of the size of the document window or the zoom factor. If the table exceeds the width of the document window, the horizontal scroll bar can be used to view the rightmost columns. **Note:** Using this option, tables may be incomplete when printed.

## Modifying the Appearance of a Table

### Table Borders

The style of exterior and interior borders is set using the **Table Properties** dialog. Select **Table>Properties**.

- You can set all, none, or only some of the borders to be visible in a table.
- You can control the visibility of interior borders by using the **Group** submenu of the **Table** menu.
- Grouping rows or columns suppresses interior borders within the table selection.
- Grouping rows and columns requires that the interior border style is set by row and column group.
- Hidden borders are visible when the mouse hovers over a table. Note that you can hide the visibility of lines on mouse pointer roll over by using the **View>Show/Hide Contents** dialog, and clearing the **Hidden Table Borders** check box.

### Alignment Options

The table alignment tools control the horizontal alignment of columns and vertical alignment of rows.

For column alignment, the current selection is expanded to encompass all rows in the selected columns. The alignment choice applies to all cells within the expanded selection. If the document does not contain a selection, the cursor position is used to identify the column.

Similarly, the selection is expanded to include all columns in the selected rows for vertical alignment options. The following table illustrates the vertical alignment options. The baseline option is useful for aligning equations across multiple cells within a row of a table.

|          |                                |                                          |
|----------|--------------------------------|------------------------------------------|
| Top      | $x \left( \frac{1}{y} \right)$ | $\frac{1}{x \left( \frac{1}{y} \right)}$ |
| Center   | $x \left( \frac{1}{y} \right)$ | $\frac{1}{x \left( \frac{1}{y} \right)}$ |
| Bottom   | $x \left( \frac{1}{y} \right)$ | $\frac{1}{x \left( \frac{1}{y} \right)}$ |
| Baseline | $x \left( \frac{1}{y} \right)$ | $\frac{1}{x \left( \frac{1}{y} \right)}$ |

### Controlling the Visibility of Cell Content

The **Table Properties** dialog includes two options to control the visibility of cell content. These options allow control over the visibility of Maple input and execution group boundaries. Thus, Maple input can be hidden in a table even if input is set to visible for the document in the **View>Show/Hide Contents** dialog.

## Printing Options

The **Table Properties** dialog contains options to control the placement of page breaks when printing. You can fit a table on a single page, allow page breaks between rows, or allow page breaks within a row.

## Execution Order Dependency

The order in which cells are executed is set in the **Table Properties** dialog. The following tables illustrate the effect of execution order.

| Row-wise execution order                                                                     |                                                                                              |
|----------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| <pre>&gt; x:=1;</pre> <p style="text-align: center;"><code>x:=1</code></p> <pre>&gt;</pre>   | <pre>&gt; x:=x+1;</pre> <p style="text-align: center;"><code>x:=2</code></p> <pre>&gt;</pre> |
| <pre>&gt; x:=x+1;</pre> <p style="text-align: center;"><code>x:=3</code></p> <pre>&gt;</pre> | <pre>&gt; x:=x+1;</pre> <p style="text-align: center;"><code>x:=4</code></p> <pre>&gt;</pre> |
| Column-wise execution order                                                                  |                                                                                              |
| <pre>&gt; x:=1;</pre> <p style="text-align: center;"><code>x:=1</code></p> <pre>&gt;</pre>   | <pre>&gt; x:=x+1;</pre> <p style="text-align: center;"><code>x:=3</code></p> <pre>&gt;</pre> |
| <pre>&gt; x:=x+1;</pre> <p style="text-align: center;"><code>x:=2</code></p> <pre>&gt;</pre> | <pre>&gt; x:=x+1;</pre> <p style="text-align: center;"><code>x:=4</code></p> <pre>&gt;</pre> |

## Tables and the Classic Worksheet

Tables are flattened on export to the Classic Worksheet interface. For example, the following table in the Standard Worksheet appears as one column in the Classic Worksheet interface.

| Table in Standard Worksheet                                                                                                                | Table in Classic Worksheet |     |     |     |     |     |                                        |
|--------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|-----|-----|-----|-----|-----|----------------------------------------|
| <table border="1"> <tr> <td>aaa</td> <td>ddd</td> </tr> <tr> <td>bbb</td> <td>eee</td> </tr> <tr> <td>ccc</td> <td>fff</td> </tr> </table> | aaa                        | ddd | bbb | eee | ccc | fff | aaa<br>ddd<br>bbb<br>eee<br>ccc<br>fff |
| aaa                                                                                                                                        | ddd                        |     |     |     |     |     |                                        |
| bbb                                                                                                                                        | eee                        |     |     |     |     |     |                                        |
| ccc                                                                                                                                        | fff                        |     |     |     |     |     |                                        |

## Examples

### Table of Values

This example illustrates how to set the visibility options for cell contents to display a table of values.

**> y := t -> 1/2\*t^2:**

| t [s]    | 0                                                    | 1                                                              | 2                                                    | 3                                                              | 4                                                 | 5                                                            | 6                                                  |
|----------|------------------------------------------------------|----------------------------------------------------------------|------------------------------------------------------|----------------------------------------------------------------|---------------------------------------------------|--------------------------------------------------------------|----------------------------------------------------|
| y(t) [m] | $\begin{array}{l} > y(0) \\ ; \\ 0 \\ > \end{array}$ | $\begin{array}{l} > y(1) \\ ; \\ \frac{1}{2} \\ > \end{array}$ | $\begin{array}{l} > y(2) \\ ; \\ 2 \\ > \end{array}$ | $\begin{array}{l} > y(3) \\ ; \\ \frac{9}{2} \\ > \end{array}$ | $\begin{array}{l} > y(4) ; \\ 8 \\ > \end{array}$ | $\begin{array}{l} > y(5) ; \\ \frac{25}{2} \\ > \end{array}$ | $\begin{array}{l} > y(6) ; \\ 18 \\ > \end{array}$ |

### Table settings:

In the **Properties** dialog (**Table>Properties** menu):

1. Set **Table Size Mode** to **Scale with zoom factor**.
2. Hide Maple input and execution group boundaries: Clear the **Show input** and **Show execution group boundaries** check boxes.

|          |   |               |   |               |   |                |    |
|----------|---|---------------|---|---------------|---|----------------|----|
| t [s]    | 0 | 1             | 2 | 3             | 4 | 5              | 6  |
| y(t) [m] | 0 | $\frac{1}{2}$ | 2 | $\frac{9}{2}$ | 8 | $\frac{25}{2}$ | 18 |

### Formatting Table Headers

The following table uses cell merging for formatting row and column headers, and row and column grouping to control the visibility of cell boundaries.

By default, invisible cell boundaries are visible on mouse pointer roll over. You can hide the visibility of lines on mouse pointer roll over by using the **View>Show/Hide Contents** dialog, and clearing the **Hidden Table Borders** check box.

|             |      | Parameter 2 |      |
|-------------|------|-------------|------|
|             |      | Low         | High |
| Parameter 1 | Low  | 13          | 24   |
|             | High | 18          | 29   |

### Table settings:

1. Insert a table with 4 rows and 4 columns.

Using the **Table** menu:

2. **Merge** the following sets of (**Row,Column**) cells: (R1,C1) to (R2,C2), (R1,C3) to (R1,C4), and (R3,C1) to (R4,C1).

3. **Group** columns 1 and 2, and columns 3 and 4.

4. **Group** rows 1 and 2, and rows 3 and 4.

In the **Properties** dialog (**Table>Properties** menu):

5. Set **Exterior Borders** to **None**.



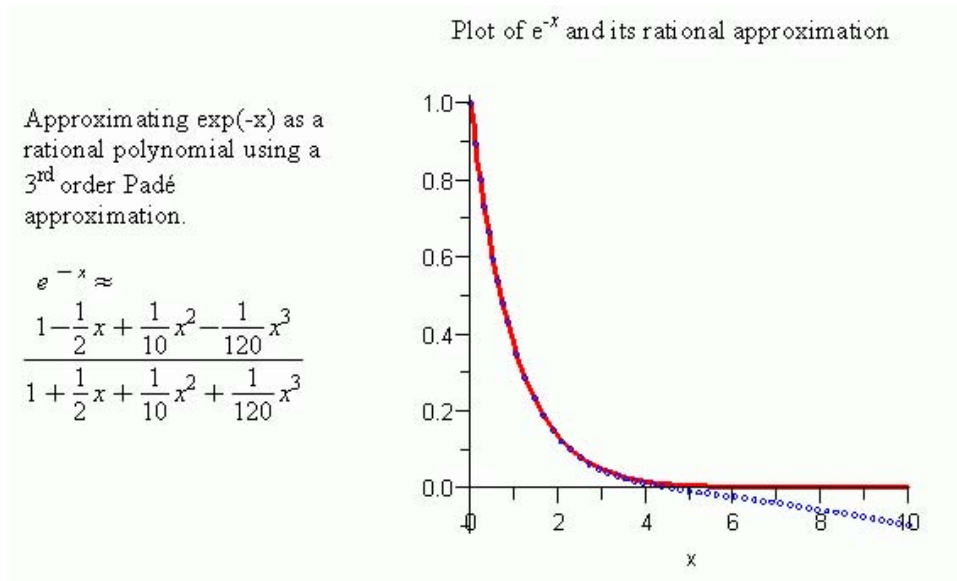
6. (Optional) Change **Table Size Mode** size option to **Scale with zoom factor**.

Using the **Table** menu:

7. Set **Alignment** of columns 3 and 4 to **Center**.

## 2-D Math and Plots

The following example illustrates the use of tables to display 2-D Math and plots side by side.



### Table Settings:

In the **Properties** dialog (**Table>Properties** menu):

1. Set **Exterior** and **Interior Borders** to **None**.
2. Hide Maple input and execution group boundaries: Clear the **Show input** and **Show execution group boundaries** check boxes.

Using the **Table** menu:

3. Change row **Alignment** to **Center**.

### Table of Mathematical Expressions

This example illustrates using the baseline alignment option to align equations across columns in a table.

| $f(x)$                                        | $\int f(x) dx$                                                                                                |
|-----------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$ | $\frac{2}{3}x - \frac{1}{9}\ln(3x + 2)$                                                                       |
| $\sin(\omega t) e^{(-at)}$                    | $-\frac{\omega e^{(-at)} \cos(\omega t)}{a^2 + \omega^2} - \frac{a e^{(-at)} \sin(\omega t)}{a^2 + \omega^2}$ |
| $\frac{d^2}{dx^2} f(x)$                       | $\frac{d}{dx} f(x)$                                                                                           |

#### Table Settings:

In the **Properties** dialog (**Table>Properties** menu):

1. Set **Exterior Border** to **Top** and **Bottom**.

Using the **Table** menu:

2. **Group** columns 1 and 2.

3. **Group** rows 2 to 4.

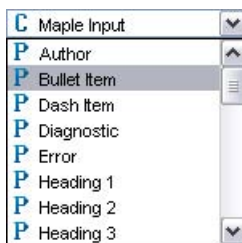
Using the **Table** menu:

4. Set row **Alignment** to **Baseline** for all rows.

## Formatting Lists: Bullets, Numbers, and Indent

Bullet, numbered, and indented lists provide an easy way to organize information in your document.

### Formatting Lists Using the Context Bar



#### To arrange content in a bullet list using the context bar drop-down list:

1. Select the text to be arranged.
2. In the character and paragraph style drop-down list, select **P Bullet Item**. The selected text is displayed as a (dot) bullet list.

#### To arrange content in a numbered list using the context bar drop-down list:

1. Select the text to be arranged.
2. In the character and paragraph style drop-down list, select **P Ordered List 1**. The selected text is displayed as a numbered list.

Ordered lists have 5 default styles. See Figure 6.12. List 1 begins at the left margin. By default, subsequent lists are indented half an inch. To change the default, see *Modifying Paragraph Styles (page 244)*. The numbering style uses numbers, lowercase letters, lowercase roman numerals, uppercase letters, uppercase Roman numerals.

- 1 Ordered List 1
  - 2
    - a Ordered List 2
      - b
        - i Ordered List 3
          - ii
            - A Ordered List 4
              - B
                - I Ordered List 5
                  - II

**Figure 6.12: Ordered List Styles**

**To arrange content in an indented list using the context bar drop-down list:**

1. Select the text to be arranged.
2. In the character and paragraph style drop-down list, select **P List Item**. The selected text is displayed as an indented list.

### **Formatting Lists Using the Paragraph Style Dialog**

With the **Paragraph Style** dialog, you can select various list styles: dot, dash, indent, numbers, lowercase letters, uppercase letters, lowercase Roman numerals, and uppercase Roman numerals.

**To arrange content in a list using the Paragraph Style dialog:**

1. Select the text to be arranged.
2. From the **Format** menu, select **Paragraph**, and then **Attributes**. The **Paragraph Style** dialog opens.
3. In the **Bullet and Numbering** drop-down list, select one of the styles.

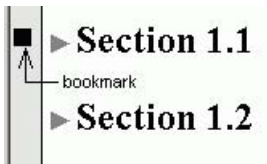
4. If you have selected one of the numbered styles (number, letters, Roman numerals), set an initial list value.
5. To continue numbering this list from a previous list in your document, select the **Linked to Previous List** check box.
6. Click **OK** to accept this style.

## Bookmarks

Use a bookmark to designate a location in an active document. This bookmark can then be accessed from other regions in your document or by using hyperlinks in other documents.

To display bookmark formatting icons, activate the **Marker** feature.

- From the **View** menu, select **Markers**.



## Inserting, Renaming, and Deleting a Bookmark

### To insert a bookmark:

1. Place the cursor at the location at which to place the bookmark.
2. From the **Format** menu, select **Bookmarks**. The **Bookmark** dialog opens, listing existing bookmarks in the document.
3. Click **New**. The **Create Bookmark** dialog opens. Enter a bookmark name and click **Create**.
4. The new bookmark appears in the **Bookmark** dialog list. Click **OK**.

**Note:** You can also rename and delete bookmarks using the **Bookmark** dialog.


## Go to a Bookmark

You can automatically move the cursor to the location of the bookmark in the active document.

1. From the **Edit** menu, select **Go To Bookmark**. The **Go To Bookmark** dialog opens with the current bookmarks listed.
2. Select the bookmark and click **OK**. The cursor moves to the bookmark.

## Inserting Images

Images help illustrate ideas and enhance presentations. You can insert images in your document at a cursor location or in a table.

|                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                     |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| <p>You can insert images in these file formats into your document.</p> <ul style="list-style-type: none"><li>• Graphics Interchange Format - gif</li><li>• Joint Photographic Experts Group - jpe, jpeg, jpg</li><li>• Portable Network Graphics - png</li><li>• Bitmap Graphics - bmp</li><li>• Tagged Image File Format - tif, tiff, jfx</li><li>• Portable aNyMap - pnm</li><li>• Kodak FlashPix - fpx</li></ul> |  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|

### To insert an image into the document at the cursor location:

1. From the **Insert** menu, select **Image**. The **Load Image** dialog opens.
2. Specify a path or folder name.
3. Select a filename.
4. Click **Open**. The image is displayed in the document.

If the source file is altered, the embedded image does not change because the original object is pasted into the document.

**To resize an inserted image:**

1. Click the image. Resizing anchors appear at the sides and corners of the image.
2. Move the mouse over the resize anchor. Resizing arrows appear.
3. Click and drag the image to the desired size.

**ImageTools Package**

You can manipulate image data using the **ImageTools** package. This package is a collection of utilities for reading and writing common image file formats, and for performing basic image processing operations within Maple.

Within Maple, images are represented as dense, rectangular Arrays of 64-bit hardware floating-point numbers. Grayscale images are 2-D, whereas color images are 3-D (the third dimension representing the color channels).

In addition to the commands in the **ImageTools** package, many ordinary Array and Matrix operations are useful for image processing.

For details about this feature, refer to the **?ImageTools** help page.

**Show or Hide Worksheet Content**

You can hide document elements of a specific type so that they are not visible. This does not delete them, but hides them from view. Hidden elements are not printed or exported, but they are copied and pasted.

In a document, use the **Show Contents** dialog to hide all spreadsheets, input, output, or graphics, plus section boundaries, execution group boundaries, and hidden table borders on mouse pointer roll over. The dialog is accessed from the **View>Show/Hide Contents** menu.

## Using the Show Contents Dialog

A check mark beside the item indicates that all document elements of that type are displayed for the current document.

1. From the **View** menu, select **Show/Hide Contents**. The **Show Contents** dialog opens with all items selected for display.
2. Clear the check box associated with the document components or ranges to hide.

By clearing the **Input** check box, only Maple Input and 2-D Math input, that is, 2-D Math content that has been evaluated, are hidden. Clearing the **Graphics** check box ensures that a plot, an image, or the **Sketch** feature inserted in the document by using the **Insert** menu option is also hidden.

## Command Output Versus Insertion

Output is considered an element that results from executing a command. Inserted components are not considered output.

Consider the following examples.

The plot resulting from executing the **plot(sin)** call is considered output.

- To show a plot from the **plot(sin)** call, select both the **Output** and **Graphics** check boxes in the **Show Contents** dialog.

If you insert a plot by using the **Insert** menu option, that plot is not considered output. Therefore, if you clear the **Output** check box in the **Show Contents** dialog, that plot will be visible in the document.

- To hide an inserted plot, clear the **Graphics** check box in the **Show Contents** dialog.

Inserted images and the **Sketch** feature are not considered output. As such, they are not hidden if you clear the **Output** check box.

- To hide an inserted image or sketch, clear the **Graphics** check box in the **Show Contents** dialog.



## 6.3 Embedded Components

You can embed simple graphical interface components, for example, a button, in your document. These components can then be associated with actions that are to be executed. For example, the value of a slider component can be assigned to a document variable, or a text field can be part of an input equation.

### Adding Graphical Interface Components

The graphical interface components can be inserted by using the **Components** palette (Figure 6.13) or by cutting/copying and pasting existing components to another area of the document. Although copied components have the same characteristics, they are distinct.

By default, palettes are displayed when you launch Maple. If palettes are not visible, use the following procedure.

#### To view palettes:

1. From the **View** menu, select **Palettes**.
2. Select **Expand Docks**.
3. If the **Components** palette is not displayed, right-click (**Control**-click, for Macintosh) the palette dock. From the context menu, select **Show Palette**, and then **Components**.

You can embed the following items.

- Button, Toggle Button
- Combo Box, Check Box, List Box
- Text Area, Label
- Slider, Plot, Function

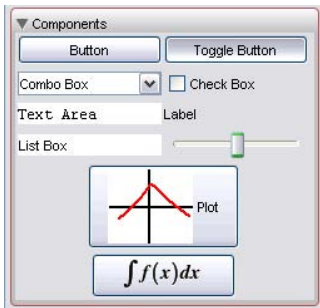


Figure 6.13: Components Palette

## Editing Component Properties: General Process

To edit properties of components embedded in the document:

1. Right-click (**Control**-click, for Macintosh) the component to display the context menu.
2. Select **Component Properties**. The related dialog opens.
3. Enter values and contents in the fields as necessary.
4. For actions, such as **Action When Value Changes** in the **Slider** component dialog, click **Edit**. A blank dialog opens allowing you to enter Maple code that is executed when the event occurs. For details, refer to the **?Document-Tools** help page.

## Removing Graphical Interface Components

You can remove an embedded component by:

- Using the **Delete** key
- Using the **Backspace** key
- Placing the cursor at the component and selecting from the document menu, **Edit>Delete Element**

## Example Component Properties

The following example inserts a slider, and a label that indicates the current value of the slider.

1. Place the cursor in the location where the embedded component is to be inserted.
2. In the **Components** palette, click the **Slider** item. A slider is inserted into the document.
3. In the **Components** palette, click the **Label** item. A label is inserted next to the slider.
4. Right-click (**Control**-click, for Macintosh) the label component. Select **Component Properties**. The **Label Properties** dialog appears.
5. Name the component **SliderLabel** and click **Ok**.
6. Right-click (**Control**-click, for Macintosh) the slider component. Select **Component Properties**. The **Slider Properties** dialog opens.
7. Name the component **Slider1**.
8. Enter the lowest position as **0** and the highest as **100**.
9. Enter minor tick marks at **10** and major tick marks at **20**.
10. To define an action when the value of the slider changes, click **Edit**. The **Action When Value Changes** dialog opens.
11. Enter the following calling sequence and click **OK** to close all dialogs.

```
DocumentTools[SetProperty ]('SliderLabel',caption,  
DocumentTools[GetProperty] ('Slider1', 'value' ) );
```

The value from the slider as you move the arrow indicator populates the **Label** caption field.

For details on these commands, refer to the **?DocumentTools/SetProperty** and the **?DocumentTools/GetProperty** help pages.

## Printing and Exporting a Document with Embedded Components

**Printing:** When printing a document, embedded components are rendered as they appear on screen.

**Exporting:** Exporting a document with embedded components to other formats produces the following results.

- HTML format - components are exported as **.gif** files.
- RTF format - components are rendered as **bitmap** images in the **.rtf** document.
- LaTeX - components are exported as **.eps** files.

## 6.4 Creating Graded Assignments

You can use Maple to create graded assignments. Question types include multiple choice, essay, true-or-false, fill-in-the-blanks, and Maple-graded.

**Note:** This feature can be used to create questions for Maple T.A.—an online automated testing and assessment system. For details about Maple T.A., see *Input, Output, and Interacting with Other Products* (page 363).

### Creating a Question

**To create a question:**

1. Open the Task browser (**Tools>Tasks>Browser**).
2. From the **Maple T.A.** folder, select the appropriate question type.
3. Insert the question template into a document.
4. Enter the question content as described in the template.
5. Repeat steps 1 to 4 for each question to add to the document.

## Viewing Questions in Maple

**To view and test your questions in Maple:**

- From the **View** menu, select **Assignment**. This view displays all of the questions in your assignment with access to hints, plotting, and grading.

After answering your questions, you can test the grading function by clicking the **Grade** button. A Maplet dialog is displayed indicating if the question was answered correctly. If hints were provided in the question, these are also displayed.

## Saving Test Content

When you save a document with test content, the authoring and assignment modes determine what the user sees when opening your document.

- If you save the document in authoring mode (task template contents visible), the user sees this content when opening the document.
- If you save the document in assignment mode, the users sees only the assignment layout.

In both cases the **View>Assignment** menu is accessible. As such, users (students) can switch between the original document contents and the displayed assignment.

## 6.5 Auto-Execute

An execution group is a grouping of Maple input with its corresponding Maple output. It is distinguished by a large square bracket, called a *group boundary*, at the left. An execution group may also contain any or all of the following: a plot, a spreadsheet, and text.

Execution groups are the fundamental computation and documentation elements in the document. If you place the cursor in an input command and press the **Enter** or **Return** key, Maple executes all of the input commands in the current execution group.

The **Autoexecute** feature allows you to designate regions of a document for automatic execution. These regions are executed when the document opens. This is useful when sharing documents. Important commands can be executed as soon as the user opens your document. The user is not required to execute all commands.

## Setting the Auto-Execute Feature

1. Select the region that must be automatically executed when the document opens.
2. From the **Format** menu, select **Autoexecute**, and then **Set**.

## Removing the Auto-Execute Setting

**To remove the setting in a region:**

1. Select the region.
2. From the **Format** menu, select **Autoexecute**, and then **Clear**.

**To remove all autoexecution in a document:**

- From the **Format** menu, select **Autoexecute**, and then **Clear All**.

## Repeating Auto-Execution

**To execute all marked groups:**

- From the **Edit** menu, select **Execute**, and then **Repeat Autoexecution**.

## Security Levels

By default, Maple prompts the user before automatically executing the document.

To set security levels for the autoexecute feature, use the **Security** tab in the **Options** dialog. For details, refer to the **?OptionsDialog** help page.

## 6.6 Sketch Regions

A sketch pad in your document allows you to quickly sketch ideas or concepts. See Figure 6.14.

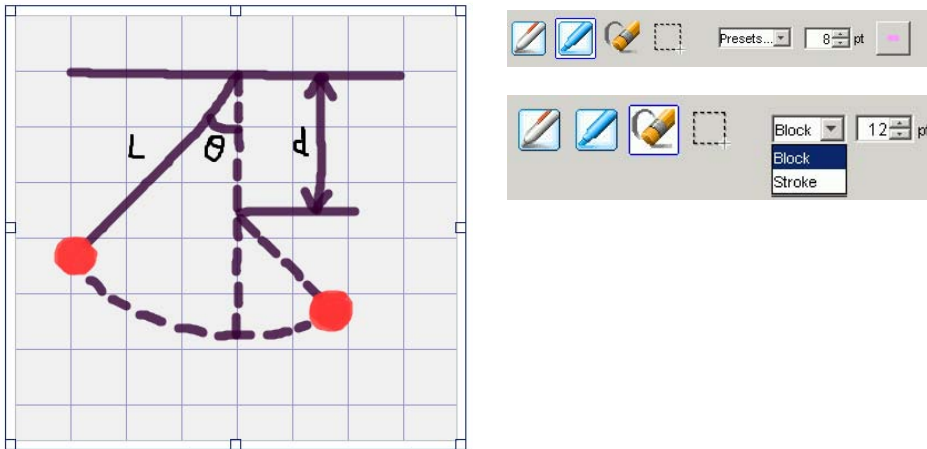


Figure 6.14: Sketch Canvas and Sketch Tools

### Insert a Sketch Pad

To insert a sketch pad:

1. Place the cursor where the sketch pad is to be inserted.
2. From the **Insert** menu, select **Sketch**. A sketch pad with grid lines appears in the document at the insertion point. The **Sketch** menu is available and associated context bar icons are displayed.

## Drawing

### To draw with a pencil or highlighter in the sketch pad:

1. From the **Sketch** menu, select **Pencil** or **Highlighter**.
2. Select a line thickness in the toolbar drop-down list.
3. With your mouse, draw lines in the sketch canvas.

### To adjust the color and width of the pencil and highlighter tools:

1. From the **Sketch** menu, select **Stroke Style Presets**. The **Stroke Styles** dialog opens.
2. Click the **Pencil** or **Highlighter** tab. The current styles are displayed.
3. To update the style, click the line to change.
4. Using the slider, alter the width and height of the line as necessary.
5. Click the **Stroke Color** button. The **Select Color** dialog opens. Choose from Swatches, HSB, and RGB colors.
6. Select a color and click **OK**. The current settings are updated with the color you selected.
7. Click **OK**.

## Canvas Style of Sketch Pad

You can alter the sketch pad (**Canvas Style**) in the following ways:

- Add a grid of horizontal and/or vertical lines. By default, the sketch opens with a grid of horizontal and vertical lines.
- Change the background color.
- Change the grid line color.
- Change the spacing between grid lines.



**To alter the canvas style:**

1. From the **Sketch** menu, select **Canvas Style**. The **Canvas Style** dialog opens.
2. For grid lines, select the appropriate grid check boxes and adjust spacing as required using the slider.
3. For colors, click the **Grid Color** or **Background Color** buttons. The **Select Color** dialog opens. Select from various colors.
4. Click **OK** to accept changes in each dialog.

**To remove gridlines:**

- Clear the **Show horizontal grid** or **Show vertical grid** check boxes, and click **OK**.

## Erase or Clear Content

**To erase individual items in the sketch pad:**

1. Click the **Eraser** icon.
2. Select the type of erasing: **Block** (point-by-point erasing) or **Stroke** (full line erasing), and eraser size.
3. Click the drawing in the sketch pad. A line (or point) is erased.

To undo this, from the **Edit** menu, select **Undo**.

**To clear the sketch pad:**

- From the **Sketch** menu, select **Clear Canvas**.

To undo this, from the **Edit** menu, select **Undo**.

## Selection Tool

The **Selection** tool allows you to select a region in the sketch pad and move the contents of that selection to another area in the sketch pad.

### To select a region:

1. From the **Sketch** menu, select **Selection**.
2. In the sketch pad, click the mouse and drag the cursor across the region to be selected.
3. Release the mouse. The area is highlighted.
4. As necessary, move the contents of the region by clicking and dragging the mouse.

## 6.7 Spell Checking

The **Spellcheck** utility examines all designated text regions of your document for potential spelling mistakes, including regions that are in collapsed sections. It does not check input, output, text in execution groups, or math in text regions. See Figure 6.15.

**Note:** The **Spellcheck** utility uses American spelling.

The CodeGeneration package is a collection of `comands` and subpackages that enable the translation of Maple code to other programming languages.

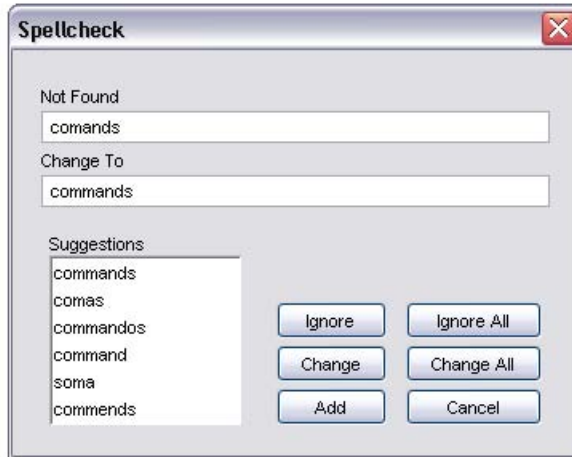


Figure 6.15: Spellcheck Dialog

## How to Use the Spellcheck Utility

1. From the **Tools** menu, select **Spellcheck**. Alternatively, press the **F7**. The **Spellcheck** dialog appears. It automatically begins checking the document for potential spelling mistakes.
2. If the **Spellcheck** utility finds a word that it does not recognize, that word is displayed in the **Not Found** text box.

You have six choices:

- To ignore the word, click **Ignore**.
- To ignore all instances of the word, click **Ignore All**.
- To change the word, that is, accept one of the suggested spellings for the word, to the one that is in the **Change To** text box, click **Change**.
- To change all instances of the word, that is, accept the suggested spelling to replace all instances of the word, click **Change All**.

- To add the word to your dictionary, click **Add**. For details, see the following **User Dictionary** section.
  - To close the **Spellcheck** dialog, that is, quit the **Spellcheck** utility, click **Cancel**.
3. When the **Spellcheck** is complete, a dialog containing the message "spellchecking complete" appears. Click **OK** to close this dialog.

## Selecting a Suggestion

To select one of the suggestions as the correct spelling, click the appropriate word from the list in the **Suggestions** text box.

If none of the suggestions are correct, highlight the word in the **Change To** text box and enter the correct spelling. Click **Change** to accept this new spelling.

## Spellcheck Usage and the Document

When using the **Spellcheck** utility, you can fix spelling errors in the **Spellcheck** dialog. You cannot change the text in the document while the **Spellcheck** utility is running.

The **Spellcheck** utility does not check grammar.

## User Dictionary

You can create and maintain a custom dictionary that works with the Maple **Spellcheck** utility.

### Properties of the Custom Dictionary File

- It must be a text file, that is, have the file extension **.txt**. For example, **mydictionary.txt**.
- It is a list of words, one word per line.
- It is case sensitive. This means that **integer** and **Integer** require individual entries in the dictionary file.

- It does not require manual maintenance. You build your dictionary file by using the **Add** functionality of the **Spellcheck**. However, you can manually edit the file if an error is introduced.

**To specify a custom dictionary to be used with the Maple Spellcheck utility:**

1. Create a **.txt** file using your favorite text editor in a directory/folder of your choice.
2. In Maple, open the **Options** dialog, **Tools>Options**, and select the **General** tab.
3. In the **User Dictionary** field, enter the path and name of the **.txt** file you created in step 1, or click **Browse** to select the location and filename.
4. To ignore Maple words that are command and function names, select the **Use Maple Words in Spellchecker** check box. A check mark indicates that the **Spellcheck** ignores Maple words.
5. Click **Apply to Session**, **Apply Globally**, or **Cancel**.

### **Adding a Word to Your Dictionary**

When running the spellcheck, if the word in the **Not Found** text box is correct, you can add the word to your dictionary.

1. Click the **Add** button. If this is the first time you are adding a word, the **Select User Dictionary** dialog opens.
2. Enter or select the custom dictionary (**.txt** file) you created. See *User Dictionary* (page 279).
3. Click **Select**. The word is automatically added to your custom dictionary file.

**Note:** Specifications in the **Options** dialog determine whether this word is recognized in your next Maple session. If you set your custom dictionary use to **Apply to Session**, then this word will *not* be recognized in a new

Maple session. If you set your custom dictionary use to **Apply Globally**, then this new word will be recognized. See *User Dictionary* (page 279).

## 6.8 Hyperlinks

Use a hyperlink in your document to access any of the following.

- Email
- Dictionary Topic
- Help Topic
- Maplet Application
- Web Page (URL)
- Document



Figure 6.16: Hyperlink Properties Dialog

### Inserting a Hyperlink in a Document

**To insert a hyperlink in the document:**

1. Position the cursor in a text region where you want to insert a hyperlink.
2. From the **Insert** menu, select **Hyperlink**.

3. In the **Hyperlink Properties** dialog box, enter the text of the hyperlink name in the **Link Text** edit field. See Figure 6.16.
4. Optionally, use an image as the link. Select the **Include an Image** check box and **Browse** for the correct file. In **.mw** files, the image appears as the link, while in **.mws** files, the **Link Text** you entered appears as the link. You can resize the image as necessary. Click the image. Resizing anchors appear at the sides and corners of the image.
5. Specify the hyperlink **Type** and **Target** as described in the appropriate following section.

### Linking to an Email Address

#### To link to an email address:

1. In the **Type** drop-down list, select **Email**.
2. In the **Target** field, enter the email address.
3. Click **OK**.

**Note:** For information about email hyperlinks in the Classic Worksheet interface, see *Worksheet Compatibility* (page 284).

### Linking to a Dictionary Topic

#### To link to a Dictionary topic:

1. In the **Type** drop-down list, select **Dictionary Topic**.
2. In the **Target** field, enter a topic name. Dictionary topics begin with the prefix **Definition/**, for example, **Definition/dimension**.
3. Click **OK**.

### Linking to a Help Page

#### To link to a help page:

1. In the **Type** drop-down list, select **Help Topic**.

2. In the **Target** field, enter the topic of the help page.

(Optional) In the **Bookmark** drop-down list, enter or select a bookmark.

3. Click **OK**.

## Linking to a Maplet Application

### To link to a Maplet application:

1. In the **Type** drop-down list, select **Maplet** .

2. In the **Target** field, enter the local path to a file with the **.maplet** extension. Optionally, click **Browse** to locate the file.

If the Maplet application exists, clicking the link launches the Maplet application. If the Maplet application contains syntax errors, then error messages are displayed in a pop-up window.

When linking to a custom Maplet application, the path is absolute. When sharing documents that contain links to Maplet applications, ensure that target Maplet applications are in the same directory.

3. Click **OK**.

**Note:** To link to a Maplet application available on a MapleNet Web page, use the URL hyperlink type to link to the Web page. For information on MapleNet, see *Input, Output, and Interacting with Other Products (page 363)*.

## Linking to a Web Page

### To link to a Web page:

1. In the **Type** drop-down list, select **URL**.

2. In the **Target** field, enter the URL, for example, **www.maplesoft.com**.

3. Click **OK**.



## Linking to a Document

### To link to a document:

1. In the **Type** drop-down list, select **Worksheet**.
2. In the **Target** field, enter the path and filename of the document or click **Browse** to locate the file. (Optional) In the **Bookmark** drop-down list, enter or select a bookmark.

**Note:** When linking to a custom document, the path is absolute. When sharing documents that contain hyperlinks, ensure that target documents are in the same directory.

3. Click **OK**.

## 6.9 Worksheet Compatibility

Maple provides users with two worksheet interfaces: the Standard Worksheet and the Classic Worksheet. Both have access to the full mathematical engine of Maple and take advantage of the new functionality in Maple. The Classic Worksheet has the traditional Maple worksheet look and uses less memory.

If you create a document in the Standard Worksheet interface of Maple and then open it in the Classic Worksheet interface, you should note possible changes to your file. For example, a bulleted list in the Standard Worksheet will not be displayed with bullets in the Classic Worksheet.

If you are creating documents for distribution, refer to the **?Compatibility** help page.

# 7 Maple Expressions

This chapter provides basic information on using Maple expressions, including an overview of the basic data structures. Many of the commands described in this chapter are useful for programming. For information on additional Maple programming concepts, such as looping, conditional execution, and procedures, see *Basic Programming* (page 321).

## 7.1 In This Chapter

| Section                                                                                                      | Topics                                                                                                                                                                                                                  |
|--------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Creating and Using Data Structures</b> - How to define and use basic data structures                      | <ul style="list-style-type: none"><li>• Expression Sequences</li><li>• Sets</li><li>• Lists</li><li>• Tables</li><li>• Arrays</li><li>• Matrices and Vectors</li><li>• Functional Operators</li><li>• Strings</li></ul> |
| <b>Working with Maple Expressions</b> - Tools for manipulating and controlling the evaluation of expressions | <ul style="list-style-type: none"><li>• Low-Level Operations</li><li>• Manipulating Expressions</li><li>• Evaluating Expressions</li></ul>                                                                              |

## 7.2 Creating and Using Data Structures

Constants, data structures, mathematical expressions, and other objects are Maple expressions. For more information on expressions, refer to the Maple Help System.

This section describes the key data structures:

- Expression sequences
- Sets
- Lists
- Tables
- Arrays
- Matrices and Vectors
- Functional operators
- Strings

## Expression Sequences

The fundamental Maple data structure is the *expression sequence*. It is a group of expressions separated by commas.

```
> S := 2, y, sin(x^2), i;
```

### Accessing Elements

**To access one of the expressions:**

- Enter the sequence name followed by the position of the expression enclosed in brackets([ ]).

For example:

```
> S[2]
```

*y*

Using negative integers, you can select an expression from the end of a sequence.

> S[-2]

$\sin(x^2)$

You can select multiple expressions by specifying a range using the range operator (..).

> S[2..-2]

$y, \sin(x^2)$

**Note:** This syntax is valid for most data structures.

## Sets

A set is an expression sequence enclosed in curly braces ({}).

>  $\left\{ 4, 12i, \sin\left(\frac{2}{3}\right) \right\}$ :

A Maple set has the basic properties of a mathematical set.

- Each element is unique. Repeated elements are stored only once.
- The order of elements is not stored.

For example:

> {c, a, a, a, b, c, a}

$\{a, c, b\}$

## Using Sets

To perform mathematical set operations, use the set data structure.

> {2, 6, 5, 1}  $\cup$  {2, 8, 6, 7}

$\{1, 2, 5, 6, 7, 8\}$ 

**Note:** The union operator is available in 1-D Math input as **union**. For more information, refer to the **?union** help page.

For more information on sets, refer to the **?set** help page.

## Lists

A list is an expression sequence enclosed in brackets ([ ]).

```
> L := [2, 3, 3, 1, 0]
```

 $L := [2, 3, 3, 1, 0]$ 

**Note:** Lists preserve both the **order** and **repetition** of elements.

### Accessing Entries

To refer to an element in a list:

- Use square brackets.

For example:

```
> L[-2..-1]
```

 $[1, 0]$ 

For more information, see *Accessing Elements* (page 286).

### Using Lists

Some commands accept a list (or set) of expressions.

For example, you can solve a list (or set) of equations using a context menu or the **solve** command.

```
> solve([x - y^2 = -2, x + y = 0])
      {y = -2, x = 2}, {y = 1, x = -1}
```

For more information, see *Solving Equations and Inequalities* (page 78).

For more information on sets and lists, refer to the **?set** help page.

## Arrays

Conceptually, the Array data structure is a generalized list. Each element has an index that you can use to access it.

The two important differences are:

- The indices can be any integers.
- The dimension can be greater than one.

### Creating and Using Arrays

To define an Array, use the **Array** constructor.

Standard **Array** constructor arguments are:

- Expression sequences of ranges - Specify the indices for each dimension
- Nested lists - Specify the contents

For example:

```
> a := Array(1..3, 1..3, [[1, 2, 3], [4, 5, 6], [7, 8, 9]])
```

$$a := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

```
> a[1, 1]
```

1

```
> a[2,3]
```

6

The **Array** constructor supports other syntaxes. It also supports many options. For more information on the **Array** constructor and the Array data structure, refer to the `?Array` help page.

## Large Arrays

Only one- and two-dimensional Arrays (with at most 10 indices in each dimension) display in the document. Larger Arrays display as a placeholder.

```
> Array(0..100)
```

```
[ 0 .. 100 Array  
  Data Type: anything  
  Storage: rectangular  
  Order: Fortran_order ]
```

### To view large Arrays:

- Double-click the placeholder.

The **Matrix Browser** displays the Array. For more information, see *Editing and Viewing Large Matrices and Vectors* (page 138).

## Tables

Tables are conceptually an extension of the Array data structure, but the table data structure is implemented using hash tables. Tables can be indexed by any values, not only integers.

## Defining Tables and Accessing Entries

> *Greek* := table([a =  $\alpha$ , b =  $\beta$ , c =  $\gamma$ ]):

> *Greek*[b]

$\beta$

You can also assign anything, for example, a list, to each element.

> *Translation* := table([one = [un, uno], two = [deux, dos], three = [trois, tres]]):

> *Translation*[two]

[deux, dos]

For more information on tables, refer to the **?table** help page.

## Matrices and Vectors

Matrices and Vectors are specialized data structures used in linear algebra and vector calculus computations.

>  $M := \begin{bmatrix} 12 & 33 \\ 83 & 12 \end{bmatrix}; v := \langle 2, 14 \rangle:$

For information on defining Matrices and Vectors, see *Creating Matrices and Vectors* (page 135).

>  $M.v$

$\begin{bmatrix} 486 \\ 334 \end{bmatrix}$



>  $v^{\%T}M$

[1186 234]

>  $M^{-1}$

$$\begin{bmatrix} \frac{-4}{865} & \frac{11}{865} \\ \frac{83}{2595} & \frac{-4}{865} \end{bmatrix}$$

For more information on these data structures, including how to access entries and perform linear algebra computations, see *Linear Algebra (page 135)*.


## Functional Operators

A functional operator is a mapping  $f: x \rightarrow y(x)$ . The value of  $f(x)$  is the result of evaluating  $y(x)$ .

Using functional operators, you can define mathematical functions.

### Defining a Function

**To define a function of one or two variables:**

1. In the **Expression** palette, click one of the function definition items. See Figure 7.1. Maple inserts the function definition.
2. Replace the placeholder **f** with the function name. Press **Tab**. **Note:** If pressing the **Tab** key inserts a tab, click the Tab icon  in the toolbar.
3. Replace the parameter placeholders, **x** or **x1**, **x2**, with the independent variable names. Press **Tab**.
4. Replace the final placeholder, **y**, with the expression that defines the function value. Press **Enter**.

**Figure 7.1: Function Definition Palette Items**

For example, define a function that adds 1 to its input.

```
> add1 := x → x + 1:
```

**Note:** To insert the right arrow, you can enter the characters `->`. In 2-D Math, Maple replaces `->` with the right arrow symbol  $\rightarrow$ . In 1-D Math, the characters are not replaced.

You can evaluate the function **add1** with symbolic or numeric arguments.

```
> add1(12); add1(x + y)
```

13

$x + y + 1$

### Distinction between Functional Operators and Other Expressions

The expression  $x + 1$  is different from the functional operator  $x \rightarrow x + 1$ .

Assign the functional operator  $x \rightarrow x + 1$  to  $f$ .

```
> f := x → x + 1:
```

Assign the expression  $x + 1$  to  $g$ .

```
> g := x + 1:
```

### To evaluate the functional operator $f$ at a value of $x$ :

- Specify the value as an argument to  $f$ .

>  $f(22)$

23

**To evaluate the expression  $g$  at a value of  $x$ :**

- You **must** use the **eval** command.

>  $eval(g, x = 22)$

23

For more information on the **eval** command, and using palettes and context menus to evaluate an expression at a point, see *Substituting a Value for a Subexpression* (page 310).

## Multivariate and Vector Functions

**To define a multivariate or vector function:**

- Enclose coordinates or coordinate functions in parentheses (( )).

For example, a multivariate function:

>  $f := (x, y) \rightarrow \frac{x^3}{y^2 + 1}$ :

>  $f(0, 0); f(-2.1, 1.9)$

0

-2.008893709

A vector function:

>  $g := t \rightarrow (\sin(t), \cos(t), t)$ :

>  $g(0); g\left(\frac{\pi}{2}\right)$

0, 1, 0

1, 0,  $\frac{1}{2}\pi$

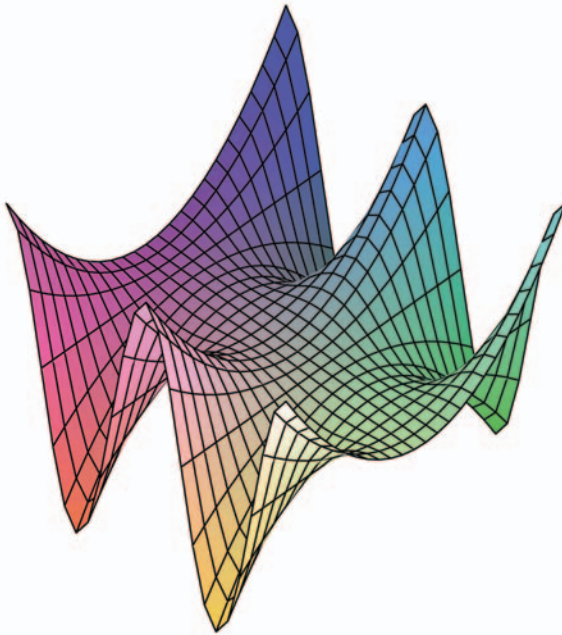
## Using Operators

To perform an operation on a functional operator, specify arguments to the operator. For example, for the operator  $f$ , specify  $f(x)$ , which Maple evaluates as an expression. See the following examples.

**Plot an Operator as an Expression** Plot a three-dimensional function using the **plot3d** command.

>  $h := (x, y) \rightarrow x^2 \cos(y)$ :

>  $plot3d(h(x, y), x = -2..2, y = -2\pi..2\pi)$



For information on plotting, see *Plots and Animations* (page 189).

**Integration** Integrate a function using the **int** command.

>  $k := x \rightarrow \sin(\cos(x)\pi)$ :

>  $\text{int}\left(k(t), t=0..\frac{\pi}{2}\right)$

$$\frac{1}{2}\pi H_0(\pi)$$

$H_0$  represents the **Struve** function. For more information, refer to the **?StruveH** help page.

For information on integration and other calculus operations, see *Calculus* (page 153).

## Strings

A string is a sequence of characters enclosed in double quotes (" ").

```
> S := "This a sequence of characters.":
```

### Accessing Characters

You can access characters in a string using brackets.

```
> S[8..-2]
```

```
"sequence of characters"
```

### Using Strings

The **StringTools** package is an advanced set of tools for manipulating and using strings.

```
> with(StringTools):
```

```
> Random(9)
```

```
"rVáIÄKJ"
```

```
> Stem("impressive")
```

```
"impress"
```

```
> Split("Create a list of strings from the words in a string")
```

```
["Create", "a", "list", "of", "strings", "from", "the", "words", "in",  
"a", "string"]
```

## 7.3 Working with Maple Expressions

This section describes how to manipulate expressions using context menus, palette items, and the underlying commands.

### To display the context menu for an expression:

- Right-click (**Control**-click, for Macintosh) the expression.

### To view the palettes:

- From the **View** menu, select **Palettes**, and then **Expand Docks**.

## Low-Level Operations

### Expression Types

A Maple *type* is a broad class of expressions that share common properties. Maple contains over 200 types, including:

- ``+``
- **boolean**
- **constant**
- **integer**
- **Matrix**
- **trig**
- **truefalse**

For more information and a complete list of Maple types, refer to the **?type** help page.

The type commands return **true** if the expression satisfies the type check. Otherwise, they return **false**.

## Testing the Type of an Expression

To test whether an expression is of a specified type:

- Use the **type** command.

```
> type(sin(x), 'trig')
```

*true*

```
> type(sin(x) + cos(x), 'trig')
```

*false*

For information on enclosing keywords in right single quotes ('), see *Delaying Evaluation* (page 317).

Maple types are not mutually exclusive. An expression can be of more than one type.

```
> type(3, 'constant')
```

*true*

```
> type(3, 'integer')
```

*true*

For information on converting an expression to a different type, see *Converting* (page 307).

## Testing the Type of Subexpressions

To test whether an expression has a subexpression of a specified type:

- Use the **hastype** command.

```
> hastype(sin(x) + cos(x), 'trig')
```



*true*

## Testing for a Subexpression

To test whether an expression contains an instance of a specified subexpression:

- Use the **has** command.

> *has*(sin(x + y), x)

*true*

> *has*(sin(x + y), x + y)

*true*

> *has*(sin(x + y), sin(x))

*false*

The **has** command searches the structure of the expression for an exactly matching subexpression.

For example, the following calling sequence returns **false**.

> *has*(x + y + z, x + z)

*false*

To return all subexpressions of a particular type, use the **indets** command. For more information, see *Indeterminates* (page 303).

## Accessing Expression Components

### Left and Right-Hand Side

The **lhs** and **rhs** commands return the left and right-hand side of an equation, inequality, or range.

#### To extract the left-hand side of an expression:

- Use the **lhs** command.

#### To extract the right-hand side of an expression:

- Use the **rhs** command.

For example:

```
> y=x + 1
```

$$y = x + 1 \tag{7.1}$$

```
> lhs((7.1))
```

$$y$$

```
> rhs((7.1))
```

$$x + 1$$

For the following equation, the left endpoint of the range is the left-hand side of the right-hand side of the equation.

```
> x=3..5
```

$$x = 3..5 \tag{7.2}$$

```
> lhs(rhs((7.2)))
```

## 3

**Numerator and Denominator****To extract the numerator of an expression:**

- Use the **numer** command.

**To extract the denominator of an expression:**

- Use the **denom** command.

$$> e := \frac{1 + \sin(x)^3 - \frac{y}{x}}{y^2 - 1 + x};$$

If the expression is not in normal form, Maple normalizes the expression before selecting the numerator or denominator. (For more information on normal form, refer to the **?normal** help page.)

> *numer(e)*

$$x + \sin(x)^3 x - y$$

> *denom(e)*

$$x(y^2 - 1 + x)$$

> *denom(denom(e))*

## 1

The expression can be any algebraic expression. For information on the behavior for non-rational expressions, refer to the **?numer** help page.

## Components of an Expression

The components of an expression are called its *operands*.

### To count the number of operands in an expression:

- Use the **nops** command.

For example, construct a list of solutions to an equation.

```
> solutions := [solve(6x^3 - x^2 + 7, x)]
solutions := [-1, 7/12 + 1/12 i sqrt(119), 7/12 + -1/12 i sqrt(119)]
```

Using the **nops** command, count the number of solutions.

```
> nops(solutions)
```

3

For more information on the **nops** command and operands, refer to the **?nops** help page.

## Indeterminates

### To find the indeterminates of an expression:

- Use the **indets** command.

The **indets** command returns the indeterminates as a set. Because the expression is expected to be rational, functions such as  $\sin(x)$ ,  $f(x)$ , and  $\text{sqrt}(x)$  are considered to be indeterminate.

```
> indets((3 + pi)x^2 sin(sqrt(1 + y)))
{y, x, sqrt(1 + y), sin(sqrt(1 + y))}
```

To return all subexpressions of a particular type, specify the type as the second argument. For information on types, see *Testing the Type of an Expression* (page 299).

```
> indets((3 + pi)x^2 sin(sqrt(1 + y)), 'radical')
      {sqrt(y + 1)}
```

To test whether an expressions has subexpressions of a specific type (without returning them), use the **has** command. For more information, see *Testing for a Subexpression* (page 300).

## Manipulating Expressions

This section introduces the most commonly used manipulation commands. For additional manipulation commands, see *Iterative Commands* (page 333).

### Simplifying

**To simplify an expression:**

- Use the **simplify** command.

The **simplify** command applies simplification rules to an expression. Maple has simplification rules for various types of expressions and forms, including trigonometric functions, radicals, logarithmic functions, exponential functions, powers, and various special functions. You can also specify custom simplification rules using a set of *side relations*.

```
> simplify(5 + 3*2 - 8*(1/3))
```

9

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2)
```

$$1 + \ln(2) + \ln(y)$$

To limit the simplification, specify the type of simplification to be performed.

$$> \text{simplify}(\sin(x)^2 + \ln(2y) + \cos(x)^2, 'trig')$$

$$1 + \ln(2y)$$

$$> \text{simplify}(\sin(x)^2 + \ln(2y) + \cos(x)^2, 'ln')$$

$$\sin(x)^2 + \ln(2) + \ln(y) + \cos(x)^2$$

You can also use the **simplify** command with *side relations*. See *Substituting a Value for a Subexpression* (page 310).

## Factoring

### To factor a polynomial:

- Use the **factor** command.

$$> \text{factor}(x^6 - x^5 - 9x^4 + x^3 + 20x^2 + 12x)$$

$$x(x - 2)(x - 3)(x + 2)(x + 1)^2$$

$$> \text{factor}(x^3y + x^2y^2 - 3x^3 - x^2y + 2xy^2 - 6x^2 - 5xy + y^2 - 3x - 3y)$$

$$(x + 1)^2(-3 + y)(x + y)$$

Maple can factor polynomials over the domain specified by the coefficients. You can also factor polynomials over algebraic extensions. For details, refer to the **?factor** help page.

For more information on polynomials, see *Polynomial Algebra* (page 126).

### To factor an integer:

- Use the **ifactor** command.

> `ifactor(196911)`

$$(3)^4 (11) (13) (17)$$

For more information on integers, see *Integer Operations (page 71)*.

## Expanding

**To expand an expression:**

- Use the **expand** command.

The **expand** command distributes products over sums and expands expressions within functions.

> `expand((y-3)(x+1)^2(x+y))`

$$x^3 y + x^2 y^2 - 3x^3 - x^2 y + 2xy^2 - 6x^2 - 5xy + y^2 - 3x - 3y$$

> `expand(sin(x+y))`

$$\sin(x) \cos(y) + \cos(x) \sin(y)$$

## Combining

**To combine subexpressions in an expression:**

- Use the **combine** command.

The **combine** command applies transformations that combine terms in sums, products, and powers into a single term.

> `combine(sin(x) cos(y) + cos(x) sin(y))`

$$\sin(x+y)$$

> `combine((xa)2 x)`

$$x^{(2a+1)}$$

The **combine** command applies only transformations that are valid for all possible values of names in the expression.

> `combine(4 ln(x) - ln(y))`

$$4 \ln(x) - \ln y$$

To perform the operation under assumptions on the names, use the **assuming** command. For more information about assumptions, see *Assumptions on Variables* (page 117).

> `combine(4 ln(x) - ln(y)) assuming x > 0, y > 0`

$$\ln\left(\frac{x^4}{y}\right)$$

## Converting

### To convert an expression:

- Use the **convert** command.

The **convert** command converts expressions to a new form, type (see *Expression Types* (page 298)), or in terms of a function. For a complete list of conversions, refer to the **?convert** help page.

Convert a measurement in radians to degrees:

> `convert(π, 'degrees')`

$$180 \text{ degrees}$$



To convert measurements that use units, use the Unit Converter or the **convert/units** command.

> `convert(450.2[[kg]], 'units', lb)`

992.5211043 [[lb]]

For information on the Unit Converter and using units, see *Units* (page 97).

Convert a list to a set:

> `convert([a, b, c, d], 'set')`

{a, b, c, d}

Maple has extensive support for converting mathematical expressions to a new function or function class.

> `convert(cos(x), exp)`

$$\frac{1}{2}e^{ix} + \frac{1}{2}e^{-ix}$$

Find an expression equivalent to the inverse hyperbolic cotangent function in terms of Legendre functions.

> `convert(arccoth(z), Legendre)`

$$Q_0\left(\frac{1}{z}\right) + \frac{1}{2} \frac{\pi \sqrt{-(z-1)^2}}{z-1}$$

$Q_0$  represents the Legendre function of the second kind. For more information, refer to the **?LegendreQ** help page.

For more information on converting to a class of functions, refer to the **?convert/to\_special\_function** help page.

## Normalizing

**To normalize an expression:**

- Use the **normal** command.

The **normal** command converts expressions into *factored normal form*.

$$> \text{normal}\left(\frac{x^2 - y^2}{(x - y)^3}\right)$$

$$\frac{x + y}{(-x + y)^2}$$

You can also use the **normal** command for zero recognition.

$$> \text{normal}(x^3 + 1 - (x + 1)^3 + 3x(1 + x))$$

$$0$$

To expand the numerator and denominator, use the **expanded** option.

$$> \text{normal}\left(\frac{x^2 - y^2}{(x - y)^3}, 'expanded'\right)$$

$$\frac{x + y}{x^2 - 2xy + y^2}$$

$$> \text{normal}\left(\sin\left(1 + \frac{1}{x}\right)\right)$$

$$\sin\left(\frac{x + 1}{x}\right)$$

## Sorting

**To sort the elements of an expression:**

- Use the **sort** command.

The **sort** command orders a list of values or terms of a polynomial.

> `sort([4, 3, 2.1, -4, 43, 0])`

`[-4, 0, 2.1, 3, 4, 43]`

> `sort(x + 4x5 - 7x2 + 1 + 9x4 - 5x3)`

`4x5 + 9x4 - 5x3 - 7x2 + x + 1`

> `sort(xy - 6y2x + 2y3 + 5x - 1)`

`2y3 - 6y2x + yx + 5x - 1`

For information on sorting polynomials, see *Sorting Terms* (page 129).

For more information on sorting, refer to the **?sort** help page.

## Evaluating Expressions

### Substituting a Value for a Subexpression

To evaluate an expression at a point, you must substitute a value for a variable.

#### To substitute a value for a variable:

1. Right-click (**Control**-click, for Macintosh) the expression. Maple displays a context menu.
2. From the context menu, select **Evaluate at a Point**. The **Evaluate at a Point** dialog is displayed.
3. In the drop-down list, select the variable to substitute.
4. In the text field, enter the value to substitute for the variable. Click **OK**.

Maple inserts the **eval** command calling sequence that performs the substitution. This is the most common use of the **eval** command.

For example, substitute  $x=3$  in the following polynomial.

$$> x^3 + 4x^2 - 7x + 2$$

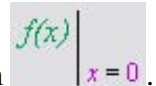
$$x^3 + 4x^2 - 7x + 2$$

$$> eval(x^3 + 4 * x^2 - 7 * x + 2, [x = 3])$$

$$44$$

**To substitute a value for a variable using palettes:**

1. In the **Expression** palette, click the evaluation at a point item
2. Specify the expression, variable, and value to be substituted.



For example:

$$> \sqrt{x^2 - x - 3} \Big|_{x=5}$$

$$\sqrt{17}$$

Substitutions performed by the **eval** function are *syntactical*, not the more powerful *algebraic* form of substitution.

If the left-hand side of the substitution is a name, Maple performs the substitution.

$$> eval(\cos(a b c), a = \frac{\pi}{6})$$

$$\cos\left(\frac{1}{6}\pi b c\right)$$

If the left-hand side of the substitution is not a name, Maple performs the substitution only if the left-hand side of the substitution is an operand of the expression.

$$> \text{eval}\left(\cos(ab), ab = \frac{\pi}{6}\right)$$

$$\frac{1}{2}\sqrt{3}$$

$$> \text{eval}\left(\cos(abc), ab = \frac{\pi}{6}\right)$$

$$\cos(abc)$$

Maple did not perform the evaluation because  $ab$  is not an operand of  $\cos(abc)$ . For information on operands, refer to the **?op** help page.

For algebraic substitution, use the **algsubs** command, or the **simplify** command with side relations.

$$> \text{algsubs}\left(ab = \frac{\pi}{6}, \cos(abc)\right)$$

$$\cos\left(\frac{1}{6}c\pi\right)$$

$$> \text{simplify}\left(\cos(abc), \left\{ab = \frac{\pi}{6}\right\}\right)$$

$$\cos\left(\frac{1}{6}c\pi\right)$$

## Numerical Approximation

**To compute an approximate numerical value of an expression:**

- Use the **evalf** command.

The **evalf** command returns a floating-point (or complex floating-point) number or expression.

$$> \text{evalf}\left(\cos\left(\frac{\pi}{6}\right)\right)$$

0.8660254040

$$> \text{evalf}\left(\frac{17}{\sqrt{3}}x^2 + x - e^\pi\right)$$

9.814954579x<sup>2</sup> + x - 23.14069264

$$> \text{evalf}(\pi)$$

3.141592654

By default, Maple calculates the result to ten digits of accuracy, but you can specify any number of digits as an index, that is, in brackets ([ ]).

$$> \text{evalf}[40](\pi)$$

3.141592653589793238462643383279502884197

For more information, refer to the **?evalf** help page.

See also *Numerically Computing a Limit* (page 155) and *Numeric Integration* (page 165).

## Evaluating Complex Expressions

To evaluate a complex expression:

- Use the **evalc** command.

If possible, the **evalc** command returns the output in the canonical form **expr1 + i expr2**.

You can enter the imaginary unit using the following two methods.

- In the **Common Symbols** palette, click the **i** or **j** item. See *Palettes (page 10)*.
- Enter *i* or *j*, and then press the symbol completion key. See *Symbol Names (page 16)*.

> `evalc( $\sqrt{1+i}$ )`

$$\frac{1}{2}\sqrt{2+2\sqrt{2}} + \frac{1}{2}i\sqrt{-2+2\sqrt{2}}$$

> `evalc(sin(3 + 5 i))`

$$\sin(3) \cosh(5) + i \cos(3) \sinh(5)$$

In 1-D Math input, enter the imaginary unit as an uppercase **i** (**I**).

> `evalc(2^(1 + I));`

$$2 \cos(\ln(2)) + 2 i \sin(\ln(2))$$

## Evaluating Boolean Expressions

To evaluate an expression involving relational operators ( = , ≠ , > , < , ≤ , and ≥ ) :

- Use the **evalb** command.

**Note:** In 1-D Math input, enter  $\neq$ ,  $\leq$ , and  $\geq$  using the  $\neq$ ,  $\leq$ , and  $\geq$  operators.

The **evalb** command uses a three-valued logic system. The return values are **true**, **false**, and **FAIL**. If evaluation is not possible, an unevaluated expression is returned.

> *evalb*( $x = x$ )

*true*

> *evalb*( $x = y$ )

*false*

> *evalb*( $3 + 2i < 2 + 3i$ )

*FAIL*

**Important:** The **evalb** command does not perform arithmetic for inequalities involving  $<$ ,  $\leq$ ,  $>$ , or  $\geq$ , and does not simplify expressions. Ensure that you perform these operations before using the **evalb** command.

> *evalb*( $\Re(x) < \Re(x + 1)$ )

$\Re(x) < 1 + \Re(x)$

> *evalb*( $\Re(x) - \Re(x + 1) < 0$ )

*true*

### Levels of Evaluation

In a symbolic mathematics program such as Maple, you encounter the issue of *levels of evaluation*. If you assign **y** to **x**, **z** to **y**, and then 5 to **z**, what is the value of **x**?



At the top-level, Maple *fully evaluates* names. That is, Maple checks if the name or symbol has an assigned value. If it has a value, Maple substitutes the value for the name. If this value has an assigned value, Maple performs a substitution, recursively, until no more substitutions are possible.

For example:

```
> x := y:
```

```
> y := z:
```

```
> z := 5:
```

Maple fully evaluates the name **x**, and returns the value 5.

```
> x
```

5

### To control the level of evaluation of an expression:

- Use the **eval** command with an integer second argument.

If passed a single argument, the **eval** command fully evaluates that expression. If you specify an integer second argument, Maple evaluates the expression to that level.

```
> eval(x)
```

5

```
> eval(x, 1)
```

y

```
> eval(x, 2)
```

$z$ 

```
> eval(x, 3)
```

 $5$ 

For more details on levels of evaluation, refer to the **?lastnameevaluation**, **?assigned**, and **?evaln** help pages.

### Delaying Evaluation

**To prevent Maple from immediately evaluating an expression:**

- Enclose the expression in right single quotes (' ').

Because right single quotes delay evaluation, they are referred to as *unevaluation quotes*.

```
> i:=4:
```

```
> i
```

 $4$ 

```
> 'i'
```

 $i$ 

### Using an Assigned Name as a Variable or Keyword

If you use an assigned name as a variable, Maple evaluates the name to its value, and passes the value to the command.

```
> 
$$\sum_{i=1}^5 i^2$$

```

Error, (in sum) summation variable previously assigned, second argument evaluates to 4 = 1 .. 5

**Note:** In general, it is recommended that you unassign a name to use it as a variable. See *Unassigning a Name Using Unevaluation Quotes* (page 319).

**To use an assigned name as a variable:**

- Enclose the name in unevaluation quotes. Maple passes the name to the command.

$$> \sum_{i=1}^5 i^2$$

55

**Important:** It is recommended that you enclose keywords in unevaluation quotes.

For example, if you enclose the keyword **left** in unevaluation quotes, Maple uses the name, not its assigned value.

> left := 3:

$$> \text{limit}\left(\frac{1}{x}, x=0, \text{'left'}\right)$$

— ∞

**Full Evaluation of an Expression in Quotes**

Full evaluation of a quoted expression removes one set of right single quotes.

> i := 4:

> ''i' + 1'

$$i + 1 \quad (7.3)$$

> (7.3)

$$i + 1 \quad (7.4)$$

> (7.4)

$$5$$

For information on equation labels and equation label references, see *Equation Labels* (page 59).

Enclosing an expression in unevaluation quotes delays evaluation, but does not prevent automatic simplification.

> 'q - i + 3 q'

$$4q - i$$

### Unassigning a Name Using Unevaluation Quotes

**To unassign a name:**

- Assign the name enclosed in unevaluation quotes to itself.

> i := 'i':

> i

$$i$$

You can also unassign a name using the **unassign** command. For more information, see *Unassigning Names* (page 57).



# 8 Basic Programming

You have used Maple interactively in the previous chapters, sequentially performing operations such as executing a single command. Because Maple has a complete programming language, you can also use sophisticated programming constructs.

**Important:** It is strongly recommended that you use the Worksheet mode and 1-D Math input when programming or using programming commands. Hence, all input in this chapter is entered as 1-D Math.

## 8.1 In This Chapter

| Section                                                        | Topics                                                                                                                                                                                                                                                                            |
|----------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Flow Control - Basic programming constructs                    | <ul style="list-style-type: none"><li>• Conditional Execution (<b>if</b> Statement)</li><li>• Repetition (<b>for</b> Statement)</li></ul>                                                                                                                                         |
| Iterative Commands - Specialized, efficient iterative commands | <ul style="list-style-type: none"><li>• Creating a sequence</li><li>• Adding and Multiplying Expressions</li><li>• Selecting Expression Operands</li><li>• Mapping a Command over a Set or List</li><li>• Mapping a Binary Command over Two Lists or Vectors</li></ul>            |
| Procedures - Maple programs                                    | <ul style="list-style-type: none"><li>• Defining and Running Simple Procedures</li><li>• Procedures with Inputs</li><li>• Procedure Return Values</li><li>• Displaying Procedure Definitions</li><li>• Displaying Maple Library Procedure Definitions</li><li>• Modules</li></ul> |

## 8.2 Flow Control

Two basic programming constructs in Maple are the **if** statement, which controls the conditional execution of statement sequences, and the **for** statement, which controls the repeated execution of a statement sequence.

### Conditional Execution (if Statement)

You can specify that Maple perform an action only if a condition holds. You can also perform an action, from a set of many, depending on which conditions hold.

Using the **if** statement, you can execute one statement from a series of statements based on a boolean (**true**, **false**, or **FAIL**) condition. Maple tests each condition in order. When a condition is satisfied, Maple executes the corresponding statement, and then exits the **if** statement.

#### Syntax

The **if** statement has the following syntax.

```
> if conditional_expression1 then  
    statement_sequence1  
elif conditional_expression2 then  
    statement_sequence2  
elif conditional_expression3 then  
    statement_sequence3  
    ...  
else  
    statement_sequenceN  
end if;
```

The conditional expressions (*conditional\_expression1*, *conditional\_expression2*, ...) can be any boolean expression. You can construct boolean expressions using:

- Relational operators - <, <=, =, >=, >, <>

- Logical operators - **and**, **or**, **xor**, **implies**, **not**
- Logical names - **true**, **false**, **FAIL**

The statement sequences (*statement\_sequence1*, *statement\_sequence2*, ..., *statement\_sequenceN*) can be any sequence of Maple statements, including **if** statements.

The **elif** clauses are optional. You can specify any number of **elif** clauses.

The **else** clause is optional.

### Simple if Statements

The simplest **if** statement has only one conditional expression.

```
> if conditional_expression then  
    statement_sequence  
end if;
```

If the conditional expression evaluates to **true**, the sequence of statements is executed. Otherwise, Maple immediately exits the **if** statement.

For example:

```
> x := 1173;
```

```
> if not isprime(x) then  
    ifactor(x);  
end if;
```

(3) (17) (23)

### else Clause

In a simple **if** statement with an **else** clause, if the evaluation of the conditional expressions returns **false** or **FAIL**, Maple executes the statement sequence in the **else** clause.



For example:

```
> if false then
    "if statement";
else
    "else statement";
end if;
```

"else statement"

## elif Clauses

In an **if** statement with **elif** clauses, Maple evaluates the conditional expressions in order until one returns **true**. Maple executes the corresponding statement sequence, and then exits the **if** statement. If no evaluation returns **true**, Maple exits the **if** statement.

```
> x := 11;
```

```
> if not type(x, integer) then
    printf("%a is not an integer.", x);
elif x >= 10 then
    printf("%a is an integer with more than one digit.", x);
elif x >= 0 then
    printf("%a is an integer with one digit.", x);
end if;
```

*11 is an integer with more than one digit.*

**Order of elif Clauses** An **elif** clause's statement sequence is executed only if the evaluation of all previous conditional expressions returns **false** or **FAIL**, and the evaluation of its conditional expression returns **true**. This means that changing the order of **elif** clauses may change the behavior of the **if** statement.

In the following **if** statement, the **elif** clauses are in the **wrong order**.

```
> if not(type(x, integer)) then
    printf("%a is not an integer.", x);
elif x >= 0 then
    printf("%a is an integer with one digit.", x);
elif x >= 10 then
    printf("%a is an integer with more than one digit.", x);
end if;
```

*11 is an integer with one digit.*

### elif and else Clauses

In an **if** statement with **elif** and **else** clauses, Maple evaluates the conditional expressions in order until one returns **true**. Maple executes the corresponding statement sequence, and then exits the **if** statement. If no evaluation returns **true**, Maple executes the statement sequence in the **else** clause.

```
> x := -12:
```

```
> if not type(x, integer) then
    printf("%a is not an integer.", x);
elif x >= 10 then
    printf("%a is an integer with more than one digit.", x);
elif x >= 0 then
    printf("%a is an integer with one digit.", x);
else
    printf("%a is a negative integer.", x);
end if;
```

*-12 is a negative integer.*

For more information on the **if** statement, refer to the **?if** help page.

## Repetition (for Statement)

Using **repetition** statements, you can repeatedly execute a statement sequence. You can repeat the statements in three ways.

- Until a counter variable value exceeds a limit (**for/from** loop)
- For each operand of an expression (**for/in** loop)
- Until a boolean condition does not hold (**while** loop)

## for/from Loop

The **for/from** loop statement repeats a statement sequence until a counter variable value exceeds a limit.

### Syntax

The **for/from** loop has the following syntax.

```
> for counter from initial by increment to final do  
    statement_sequence  
end do;
```

The behavior of the **for/from** loop is:

1. Assign the *initial* value to the name **counter**.
2. Compare the value of **counter** to the value of *final*. If the **counter** value **exceeds** the *final* value, exit the loop. (This is the *loop bound test*.)
3. Execute the *statement\_sequence*.
4. Increment the **counter** value by the value of *increment*.
5. Repeat steps 2 to 4, until Maple exits the loop.

The **from**, **by**, and **to** clauses are optional and can be in any order between the **for** clause and the **do** keyword.

Table 8.1 lists the default clause values.

**Table 8.1: Default Clause Values**

| Clause                     | Default Value                |
|----------------------------|------------------------------|
| <b>from</b> <i>initial</i> | 1                            |
| <b>by</b> <i>increment</i> | 1                            |
| <b>to</b> <i>final</i>     | <b>infinity</b> ( $\infty$ ) |

## Examples

The following loop returns the square root of the integers 1 to 5 (inclusive).

```
> for n to 5 do
    evalf(sqrt(n));
end do;
```

1.

1.414213562

1.732050808

2.

2.236067977

When the value of the counter variable **n** is **strictly greater than 5**, Maple exits the loop.

```
> n;
```

6

The previous loop is equivalent to the following **for/from** statement.

```
> for n from 1 by 1 to 5 do
    evalf(sqrt(n));
end do;
```

The **by** value can be negative. The loop repeats until the value of the counter variable is **strictly less than** the **final** value.

```
> for n from 10 by -1 to 3 do
    if isprime(n) then
        print(n);
    end if;
end do;
```

7

5

3

```
> n;
```

2

## for/in Loop

The **for/in** loop statement repeats a statement sequence for each component (*operand*) of an expression, for example, the elements of a list.

### Syntax

The **for/in** loop has the following syntax.

```
> for variable in expression do
    statement_sequence
end do;
```

The **for** clause must appear first.

The behavior of the **for/in** loop is:

1. Assign the first operand of *expression* to the name *variable*.
2. Execute the *statement\_sequence*.
3. Assign the next operand of *expression* to *variable*.
4. Repeat steps 2 and 3 for each operand in *expression*. If there are no more operands, exit the loop. (This is the *loop bound test*.)

### Example

The following loop returns a floating-point approximation to the **sin** function at the angles (measured in degree) in the list **L**.

```
> L := [23.4, 87.2, 43.0, 99.7]:
```

```
> for i in L do  
    evalf(sin(i*Pi/180));  
end do;
```

```
0.3971478907
```

```
0.9988061374
```

```
0.6819983602
```

```
0.9857034690
```

### while Loop

The **while** loop repeats a statement sequence until a boolean expression does not hold.

## Syntax

The **while** loop has the following syntax.

```
> while conditional_expression do  
    statement_sequence  
end do;
```

A **while** loop repeats until its boolean expression *conditional\_expression* evaluates to **false** or **FAIL**. For more information on boolean expressions, see *Conditional Execution (if Statement)* (page 322).

## Example

The following loop computes the digits of 872, 349 in base 7 (in order of *increasing* significance).

```
> x := 872349:
```

```
> while x > 0 do  
    irem(x, 7);  
    x := iquo(x, 7);  
end do;
```

2

x := 124621

0

x := 17803

2

x := 2543

```
2
x := 363
6
x := 51
2
x := 7
0
x := 1
1
x := 0
```

To perform such conversions efficiently, use the **convert/base** command.

```
> convert(872349, base, 7);
```

```
[2, 0, 2, 2, 6, 2, 0, 1]
```

For information on non-base 10 numbers, see *Non-Base 10 Numbers* (page 74).

### General Loop Statements

You can include a **while** statement in a **for/from** or **for/in** loop.

The general **for/from** loop has the following syntax.



```
> for counter from initial by increment to final  
  while conditional_expression do  
    statement_sequence  
  end do;
```

The general **for/in** loop has the following syntax.

```
> for variable in expression  
  while conditional_expression do  
    statement_sequence  
  end do;
```

After testing the loop bound condition at the beginning of each iteration of the **for** loop, Maple evaluates *conditional\_expression*.

- If *conditional\_expression* evaluates to **false** or **FAIL**, Maple exits the loop.
- If *conditional\_expression* evaluates to **true**, Maple executes *statement\_sequence*.

## Infinite Loops

You can construct a loop for which there is no exit condition, for example, a **while** loop in which the *conditional\_expression* always evaluates to **true**. This is called an *infinite loop*. Maple indefinitely executes an infinite loop unless it executes a **break**, **quit**, or **return** statement or you interrupt the computation. For more information, refer to the **?break**, **?quit**, **?return**, and **?interrupt** help pages.

## Additional Information

For more information on the **for** statement and looping, refer to the **?do** help page.

## 8.3 Iterative Commands

Maple has commands that perform common selection and repetition operations. These commands are more efficient than similar algorithms implemented using library commands. Table 8.2 lists the iterative commands.

**Table 8.2: Iterative Commands**

| Command             | Description                                                                                             |
|---------------------|---------------------------------------------------------------------------------------------------------|
| <b>seq</b>          | Create sequence                                                                                         |
| <b>add</b>          | Compute numeric sum                                                                                     |
| <b>mul</b>          | Compute numeric product                                                                                 |
| <b>select</b>       | Return operands that satisfy a condition                                                                |
| <b>remove</b>       | Return operands that do not satisfy a condition                                                         |
| <b>selectremove</b> | Return operands that satisfy a condition and separately return operands that do not satisfy a condition |
| <b>map</b>          | Apply command to the operands of an expression                                                          |
| <b>zip</b>          | Apply binary command to the operands of two lists or vectors                                            |

### Creating a Sequence

The **seq** command creates a sequence of values by evaluating a specified expression over a range of index values or the operands of an expression. See Table 8.3.

**Table 8.3: The seq Command**

| Calling Sequence Syntax                                                            | Examples                                                  |
|------------------------------------------------------------------------------------|-----------------------------------------------------------|
| <code>seq(<i>expression</i>, <i>name</i> = <i>initial</i> .. <i>final</i>);</code> | <pre>&gt; seq(exp(x), x=-2..0);</pre> $e^{-2}, e^{-1}, 1$ |

| Calling Sequence Syntax                           | Examples                                                                                              |
|---------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| <code>seq(expression, name in expression);</code> | <p>&gt; <b>seq(u, u in [Pi/4, Pi^2/2, 1/Pi]);</b></p> $\frac{\pi}{4}, \frac{\pi^2}{2}, \frac{1}{\pi}$ |

## Adding and Multiplying Expressions

The **add** and **mul** commands add and multiply sequences of expressions over a range of index values or the operands of an expression. See Table 8.4.

**Table 8.4: The add and mul Commands**

| Calling Sequence Syntax                                                                                          | Examples                                                                                                                               |
|------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------|
| <code>add(expression, name = initial .. final);</code><br><code>mul(expression, name = initial .. final);</code> | <p>&gt; <b>add(exp(x), x = 2..4);</b></p> $e^2 + e^3 + e^4$ <p>&gt; <b>mul(2*x, x = 1 .. 10);</b></p> $3715891200$                     |
| <code>add(expression, name in expression);</code><br><code>mul(expression, name in expression);</code>           | <p>&gt; <b>add(u, u in [Pi/4, Pi/2, Pi]);</b></p> $\frac{7\pi}{4}$ <p>&gt; <b>mul(u, u in [Pi/4, Pi/2, Pi]);</b></p> $\frac{\pi^3}{8}$ |

The endpoints of the index range (**initial** and **final**) in the **add** and **mul** calling sequence must evaluate to numeric constants. For information on symbolic sums and products, refer to the **?sum** and **?product** help pages.

## Selecting Expression Operands

The **select**, **remove**, and **selectremove** commands apply a boolean-valued procedure or command to the operands of an expression. For information on operands, refer to the **?op** help page.

- The **select** command returns the operands for which the procedure or command returns **true**.
- The **remove** command returns the operands for which the procedure or command returns **false**.
- The **selectremove** command returns two expressions of the same type as the input expression.

- The first consists of the operands for which the procedure or command returns **true**.

- The second consists of the operands for which the procedure or command returns **false** or **FAIL**.

See Table 8.5.

For information on Maple procedures, see *Procedures* (page 338).

**Table 8.5: The select, remove, and selectremove Commands**

| Calling Sequence Syntax                    | Examples                                                                               |
|--------------------------------------------|----------------------------------------------------------------------------------------|
| <code>select(proc_cmd, expression);</code> | <pre>&gt; select(issqr, {198331, 889249, 11751184, 9857934}); {889249, 11751184}</pre> |
| <code>remove(proc_cmd, expression);</code> | <pre>&gt; remove(var -&gt; degree(var) &gt; 3, 2*x^3*y - y^3*x + z); z</pre>           |

| Calling Sequence Syntax                          | Examples                                                                                                                       |
|--------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|
| <code>selectremove(proc_cmd, expression);</code> | <pre>&gt; selectremove(x -&gt; evalb(x &gt; round(x)), [sin(0.), sin(1.), sin(3.)]);  [0.1411200081], [0., 0.8414709848]</pre> |

For information on optional arguments to the selection commands, refer to the `?select` help page.

## Mapping a Command over a Set or List

The `map` command applies a name, procedure, or command to each element in a set or list. See Table 8.6.

**Table 8.6: The `map` Command**

| Calling Sequence Syntax                      | Examples                                                                                                                                                                                                                           |
|----------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>map(name_proc_cmd, expression);</code> | <pre>&gt; map(f, {a, b, c});  {f(a), f(c), f(b)}</pre> <pre>&gt; map(u -&gt; int(cos(x), x = 0 .. u), [Pi/4, Pi/7, Pi/3.0]);  <math>\left[ \frac{\sqrt{2}}{2}, \cos\left(\frac{5\pi}{14}\right), 0.8660254038 \right]</math></pre> |

For information on mapping over the operands of other expressions, optional arguments to the `map` command, and other mapping commands, refer to the `?map` help page.

## Mapping a Binary Command over Two Lists or Vectors

The `zip` command applies a name or binary procedure or command component-wise to two lists or vectors.

By default, the length of the returned object is that of the shorter list or vector. If you specify a value as the (optional) fourth argument, it is used as the value of the missing elements of the shorter list or vector. In this case, the length of the return value is that of the longer list or vector. See Table 8.7.

**Table 8.7: The zip Command**

| Calling Sequence Syntax                                                      | Examples                                                                                                                                                                                        |
|------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>zip(proc_cmd, a, b);</code><br><code>zip(proc_cmd, a, b, fill);</code> | <pre>&gt; zip(f, [i, j], [k, l]);</pre> $[f(i, k), f(j, l)]$ <pre>&gt; zip(AiryAi, [1, 2], [0], 1);</pre> $\left[ -\frac{3^{(1/6)} \Gamma\left(\frac{2}{3}\right)}{2\pi}, \text{Ai}(1) \right]$ |

For more information on the **zip** command, refer to the **?zip** help page.

## Additional Information

For more information on looping commands, refer to the corresponding command help page.

## 8.4 Procedures

A Maple procedure is a program consisting of Maple statements. Using procedures, you can quickly execute the contained sequence of statements.

### Defining and Running Simple Procedures

To define a procedure, enclose a sequence of statements between **proc(...)** and **end proc** statements. In general, you assign a procedure definition to a name.

The following procedure returns the square root of 2.

```
> p := proc() sqrt(2); end proc;
```

```
p := proc () sqrt(2) end proc
```

**Note:** Maple returns the procedure definition.

To improve readability of procedures, it is recommended that you define a procedure using multiple lines, and indent the lines using space characters. To begin a new line (without evaluating the incomplete procedure definition), press **Shift+Enter**. When you have finished entering the procedure, press **Enter** to create the procedure.

For example:

```
> p := proc()  
    sqrt(2);  
end proc;
```

To run the procedure **p**, enter its name followed by parentheses (( )).

```
> p();
```

$\sqrt{2}$

## Procedures with Inputs

You can define a procedure that accepts user input. In the parentheses of the **proc** statement, specify the parameter names. For multiple parameters, separate the names with commas.

```
> geometric_mean := proc(x, y)
    sqrt(x*y);
end proc;
```

When the user runs the procedure, the parameter names are replaced by the argument values.

```
> geometric_mean(13, 17);
```

$$\sqrt{221}$$

```
> geometric_mean(13.5, 17.1);
```

15.19374871

For more information on writing procedures, including options and local and global variables, refer to the **?procedure** help page.

## Procedure Return Values

When you run a procedure, Maple returns **only** the last statement result value computed. Maple does not return the output for each statement in the procedure. It is irrelevant whether you use semicolons or colons as statement separators.

```
> p := proc(a, b)
    a + b;
    a - b;
end proc;
```



```
> p(1, 2);
```

-1

## Displaying Procedure Definitions

Unlike simple Maple objects, you cannot display the value of a procedure by entering its name.

```
> geometric_mean;
```

*geometric\_mean*

You must evaluate the name of the procedure using the **print** (or **eval**) command.

```
> print(geometric_mean);
```

```
proc (x,y) sqrt(x*y) end proc
```

## Displaying Maple Library Procedure Definitions

Maple procedure definitions are a valuable learning tool. To learn how to program in Maple, it is recommended that you examine the procedures available in the Maple library.

By default, the **print** command returns only the **proc** and **end proc** statements and (if present) the description fields of a Maple procedure.

```
> print(assign);
```

```
proc (a) ...end proc
```

To display a Maple library procedure definition, first set the value of the **interface verboseproc** option to **2**. Then re-execute the **print** calling sequence. See Figure 8.1.

> **interface('verboseproc' = 2):**

> **print(assign);**

```

proc(a)
  option Copyright (c) 1990 by Waterloo Maple Inc. All rights reserved.;
  local i;
  if 1 <= nargs and type(a, {'::', 'name', 'function'}) then
    a := args[2..-1]
  elif nargs = 1 then
    if type(a, {'::', 'name', 'function'}) = 'anything' then
      `assign/internal`(op(a))
    elif type(a, '=') then
      if type([lhs(a)], ('list')({'::', 'name', 'function'})) then
        if nops([lhs(a)]) = nops([rhs(a)]) then
          zip('assign/internal', [lhs(a)], [rhs(a)])
        else
          error "ambiguous multiple assignment"
        end if
      else
        error "invalid arguments"
      end if
    elif type(a, ('list', 'set')) then
      map(procname, a)
    else
      error "invalid arguments"
    end if
  else
    seq(procname(i), i = args)
  end if;
  NULL
end proc

```

**Figure 8.1: Displaying assign Procedure**

## Modules

Maple procedures associate a sequence of commands with a single command. The module, a more complex programming structure, allows you to associate related procedures and data.

A key feature of modules is that they *export* variables. This means that the variables are available outside the module in which they are created. Most Maple packages are implemented as modules. The package commands are exports of the module.

For more information on modules, refer to the **?module** help page.

# 9 Maplets

A Maplet is a graphical user interface that provides interactive access to the Maple engine through buttons, text regions, slider bars, and other visual interfaces. You can design custom Maplet applications to use and share with colleagues or students, or you can take advantage of the built-in Maplets that cover numerous academic and specialized topics. For information on some of the built-in Maplets, see *Assistants and Tutors* (page 26), *Teaching and Learning with Maple* (page 180), or *Ordinary Differential Equations (ODEs)* (page 89). Other methods of interaction with Maple are described in the *Maple Getting Started Guide* and throughout this book.

## 9.1 In This Chapter

| Section                                                       | Topics                                                                                                             |
|---------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| Simple Maplet - Illustrating a simple Maplet                  | <ul style="list-style-type: none"><li>• Define and Run a Simple Maplet</li></ul>                                   |
| Using Maplets - Methods for launching a Maplet                | <ul style="list-style-type: none"><li>• Maplet File</li><li>• Maple Document</li></ul>                             |
| Authoring Maplets - Methods for authoring and saving a Maplet | <ul style="list-style-type: none"><li>• Maplet Builder</li><li>• <b>Maplets</b> Package</li><li>• Saving</li></ul> |

## 9.2 Simple Maplet

A Maplet application can be defined using the commands in the **Maplets[Elements]** package and then launched using the **Maplets[Display]** command. The following commands define and run a very simple Maplet application that contains the text string "Hello World".

> **with(Maplets[Elements]):**

```
> MySimpleMaplet:= Maplet([[ "Hello World" ]]):
```

```
> Maplets[Display](MySimpleMaplet):
```



**Figure 9.1: A Simple Maplet**

For more information on creating Maplets, including an overview of the point-and-click **Maplet Builder Assistant**, see *Authoring Maplets* (page 345).

## 9.3 Using Maplets

Maplet applications are launched by executing Maplet code. Maplet code can be saved in a Maplet (**.maplet**) file or Maple document (**.mw**).

### Maplet File

To launch a Maplet application saved as a Maplet file:

- In Windows, double-click the file from a Windows file browser.
- In UNIX and on Macintosh, use the command-line interface. At the command-line, enter **maple(maplet\_filename)**.

To view and edit the Maplet code contained within the **.maplet** file:

1. Start Maple.
2. From the **File** menu, select **Open**. Maple displays the **Open** dialog.
3. In the **Files of Type** drop-down list, select **.maplet**.
4. Navigate to the location of the **.maplet** file and select the file.
5. Click **Open**.

## Maple Document

To launch a Maplet application for which the Maple code is contained in a Maple document, you need to execute the Maplet code. To display the Maplet application, you must use the **Maplets[Display]** command. **Note:** The Maplet code may be quite large if the Maplet application is complex. In this case, execute the document to ensure user-defined procedures that are referenced in the Maplet application are also defined.

### Typical procedure:

1. If present, evaluate user-defined procedures.

```
Myproc:=proc...
```

2. Load the **Maplets[Elements]** package.

```
with( Maplets[Elements] );
```

3. Evaluate the Maplet definition.

```
Maplet_name:=Maplet( Maplet_definition );
```

4. Display the Maplet application.

```
Maplets[Display]( Maplet_name );
```

**Important:** When a Maplet application is running, you cannot interact with the Maple document.

## 9.4 Authoring Maplets

When authoring Maplets, you can use the **Maplet Builder** (GUI-based) or the **Maplets** package (syntax-based). The **Maplet Builder** allows you to drag and drop buttons, sliders, text regions, and other elements to define the Maplet application and set the element properties to perform an action upon selection or update of the element. The **Maplet Builder** is designed to create

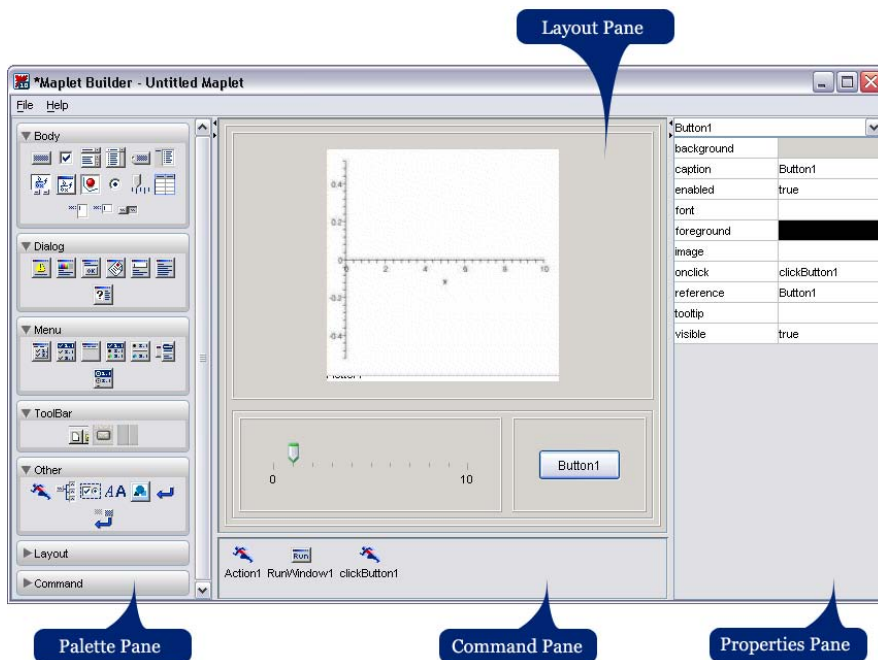
simple Maplets. The **Maplets** package offers more capabilities, control and options when designing complicated Maplet applications.

Designing a Maplet application is similar to constructing a house. When building a house, you first construct the skeletal structure (that is, foundation, floors, and walls) and then proceed to add the windows and doors. Constructing a Maplet is no different. First define the rows and columns of the Maplet application and then proceed to add the body elements (for example, buttons, text fields, and plotter regions).

## Maplet Builder

To start the **Maplet Builder**:

- From the **Tools** menu, select **Assistants**, and then **Maplet Builder**.



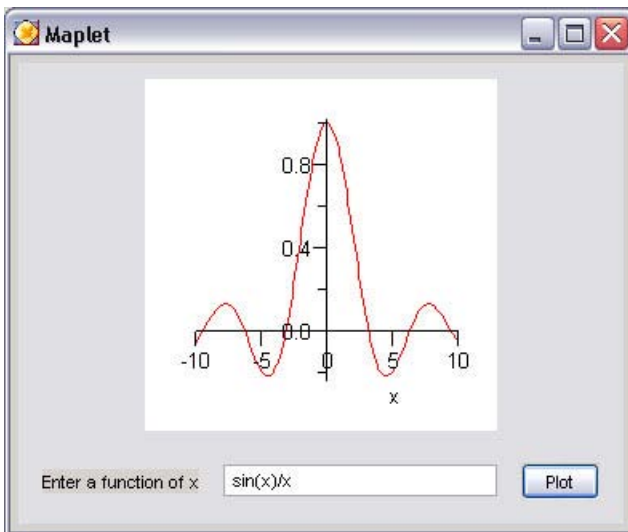
**Figure 9.2: Maplet Builder Interface**

The **Maplet Builder** is divided into four different panes.

- The **Palette** pane displays palettes, which contain Maplet elements, organized by category. For a description of the elements, see the **?MapletBuilder/Palette** help page. The **Body** palette contains the most popular elements.
- The **Layout** pane displays the visual elements of the Maplet.
- The **Command** pane displays the commands and corresponding actions defined in the Maplet.
- The **Properties** pane displays the properties of an instance of a defined element in the Maplet.

### Design a Maplet Using the Maplet Builder

In this example, shown in Figure 9.3, the Maplet user enters a function and plots the result.



**Figure 9.3: Image of the Maplet**



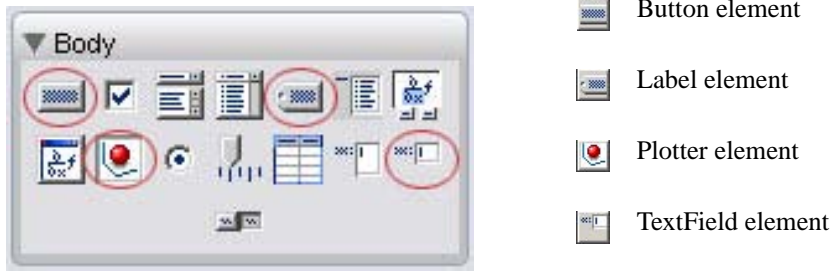


Figure 9.4: Body Elements Used When Defining This Maplet

## Define the number of rows in the Maplet

1. In the **Properties** pane:

- a. In the drop-down list, select **BoxColumn1**.
- b. Change the **numrows** field to **2**.

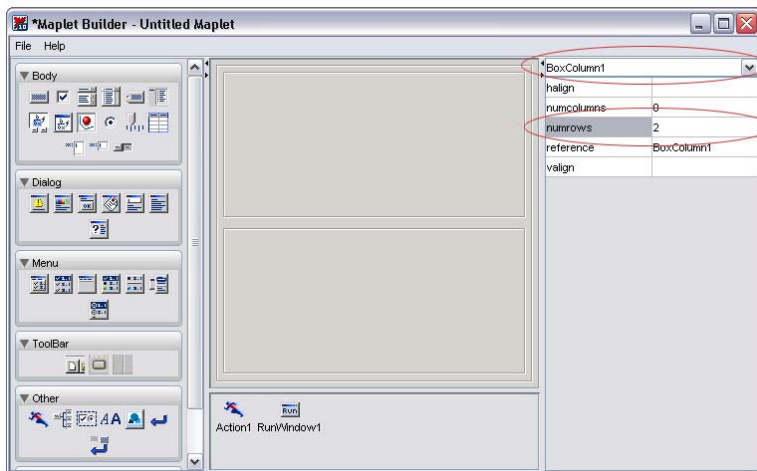


Figure 9.5: Define the Number of Rows in the Maplet

## Add a plot region to row 1

1. From the **Body** palette, drag the **Plotter** element to the first row in the **Layout** pane.

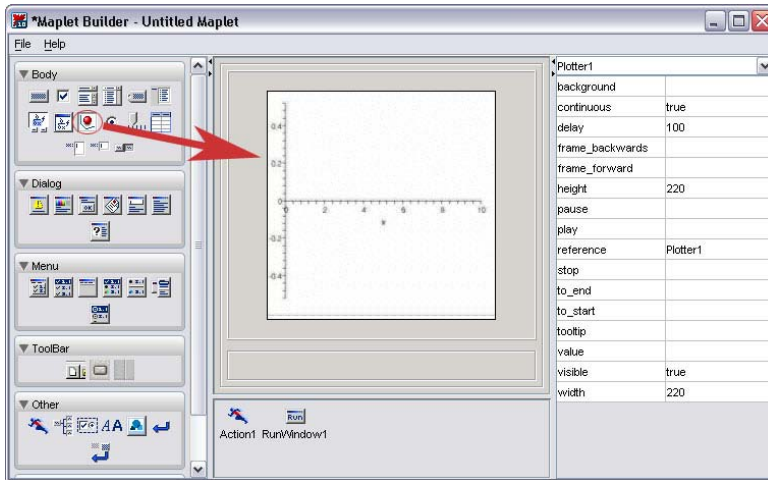


Figure 9.6: Add a Plot to Row 1

## Add columns to row 2

1. In the **Properties** pane:
  - a. In the drop-down list, select **BoxRow2**.
  - b. Change the **numcolumns** field to **3**.

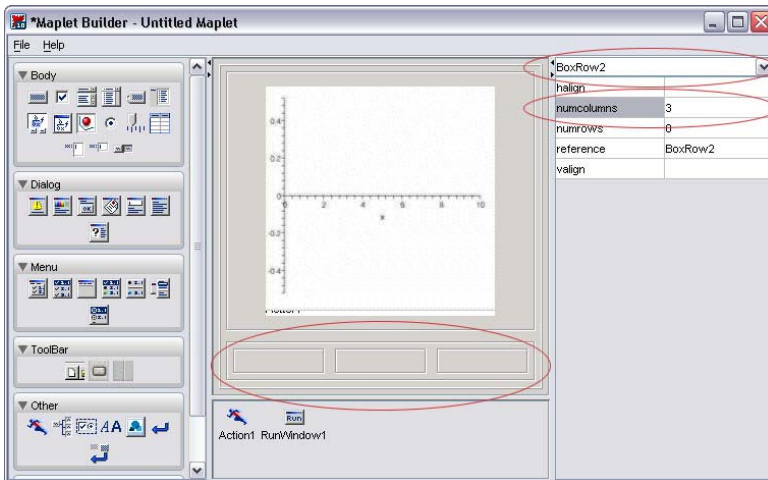


Figure 9.7: Add Columns to Row 2

## Add a label to row 2

1. From the **Body** palette, drag the **Label** element to the left column in the **Layout** pane.
2. In the **Properties** pane:
  - a. In the drop-down list, select **Label1**.
  - b. Change the **caption** field to **Enter a function of x**.

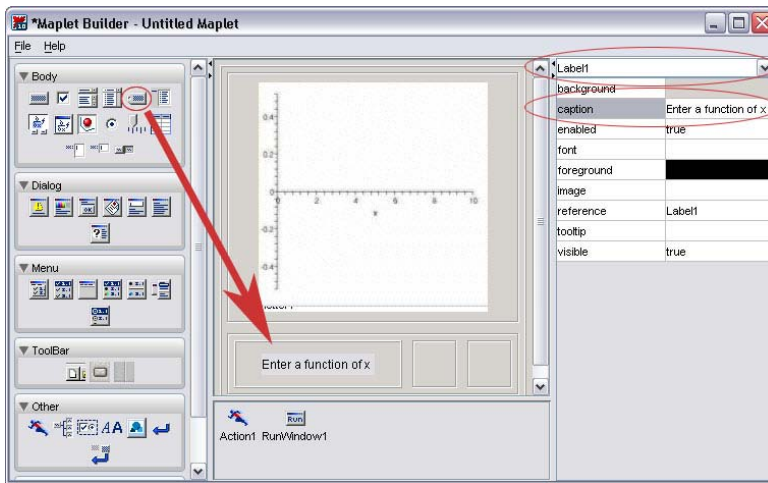


Figure 9.8: Add a Label to Row 2

## Add a text region to row 2

1. From the **Body** palette, drag the **TextField** element to the middle column. The **TextField** element allows the Maplet user to enter input that can be retrieved in an action.
2. If necessary, resize the Maplet Builder to display the entire **Layout** pane.

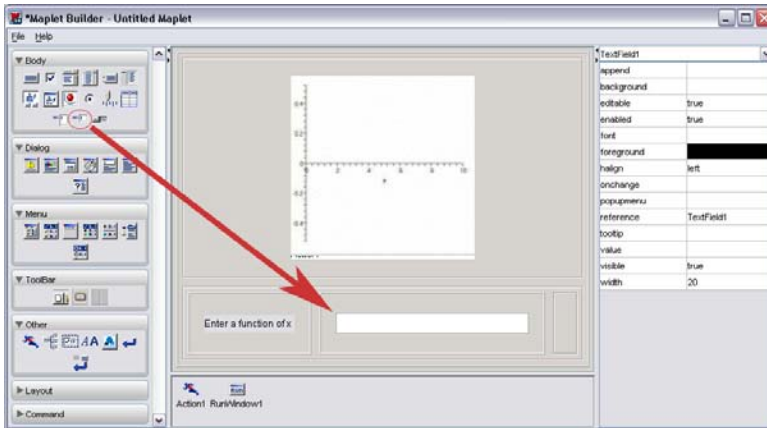


Figure 9.9: Add a Text Region to Row 2

## Add a button to row 2

1. From the **Body** palette, drag the **Button** element to the right column in the **Layout** pane.
2. In the **Properties** pane:
  - a. In the drop-down list, select **Button1**.
  - b. Change the **caption** field to **Plot**.
  - c. In the **onclick** property drop-down list, select **<Evaluate>**.

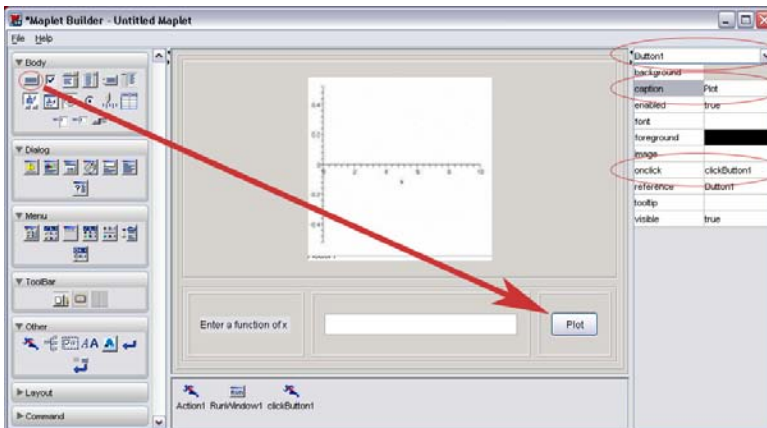


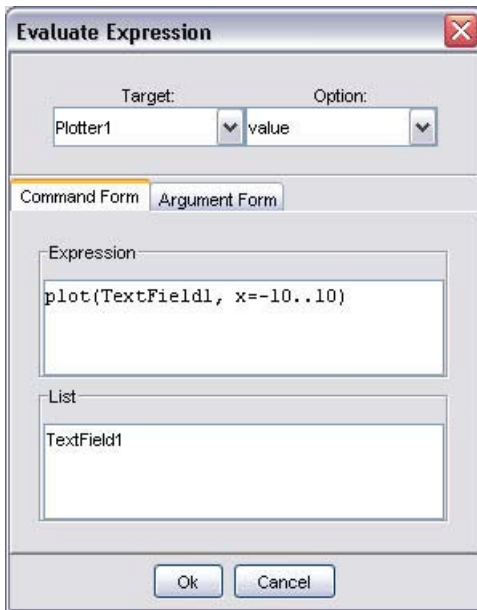
Figure 9.10: Add a Button to Row 2

3. In the **Evaluate Expression** dialog that displays, the **Target** drop-down list contains the defined elements to which you can send information, in this case, **Plotter1** and **TextField1**. The **List** group box, located below the **Expression** group box, displays the defined elements to which you can retrieve information, in this case, **TextField1**.

a. In the **Target** drop-down list, select **Plotter1**.

b. In the **Command Form** tab, enter **plot(TextField1, x=-10..10)** in the **Expression** group box. (**Note:** Do not include a semicolon (;) at the end of the plot command). You can also double-click **TextField1** in the **List** group box to insert this element in the command syntax.

c. Click **Ok**.



**Figure 9.11: Evaluate Expression Dialog**

## Run the Maplet

1. From the **File** menu, select **Run**. You are prompted to save the Maplet. Maplets created with the **Maplet Builder** are saved as **.maplet** files.
2. Click **Yes** and navigate to a location to save this Maplet.

For further information on the **Maplet Builder**, see the **?MapletBuilder** help page. For more examples of designing Maplets using the **Maplet Builder**, see **?MapletBuilder/examples**.

## Maplets Package

When designing a complicated Maplet, the **Maplets** package offers greater control. The **Maplets[Elements]** subpackage contains the elements available when designing a Maplet application. After you define the Maplet, use the **Maplets[Display]** command to launch the Maplet.



### Example 1 - Design a Maplet Using the Maplets Package

To introduce the structure of designing Maplets using the **Maplets** package, this example illustrates the equivalent syntax for the *Design a Maplet Using the Maplet Builder* (page 347).

Load the **Maplets[Elements]** package.

**> with(Maplets[Elements]):**

Define the Maplet application. To suppress the display of the data structure associated with the Maplet application, end the definition with a colon.

```
> PlottingMaplet:=Maplet(  
  BoxLayout(  
    BoxColumn(  
      # First Box Row  
      BoxRow(  
        # Define a Plot region  
        Plotter('reference' = Plotter1)  
      # End of first Box Row  
      ),  
      # Second Box Row  
      BoxRow(  
        # Define a Label  
        Label("Enter a function of x "),  
        # Define a Text Field  
        TextField('reference' = TextField1),  
        # Define a Button  
        Button(caption="Plot", Evaluate(value = 'plot(TextField1,  
          x = -10..10)', 'target' = Plotter1))  
      # End of second Box Row  
      )  
    # End of BoxColumn  
    )  
  # End of BoxLayout  
  )  
# End of Maplet  
):
```

Launch the Maplet.

```
> Maplets[Display](PlottingMaplet);
```

## Example 2 - Accessing User-Defined Procedures

When designing a Maplet, you can access user-designed procedures and send information bi-directionally to the Maplet. In this example, shown in Figure 9.12, the user enters a function in a MathML editor region, optionally selects a color from a color dialog, and plots the result.

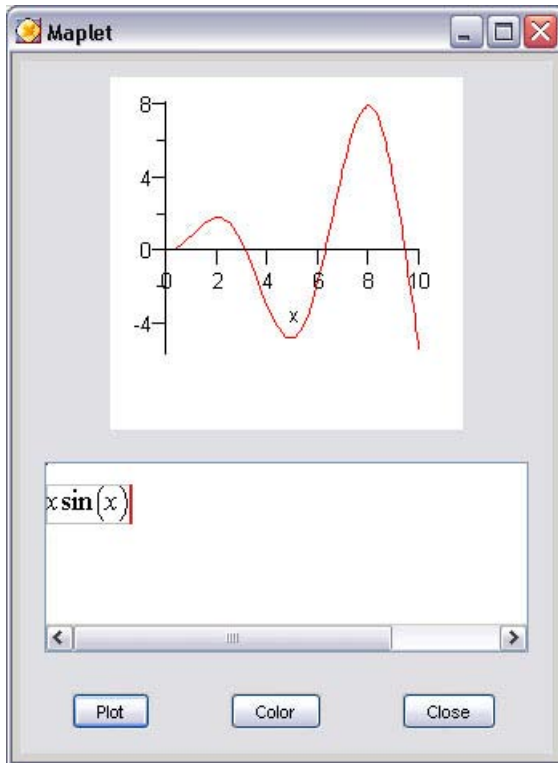


Figure 9.12: Image of the Maplet

## User-Defined Procedure and Maplet Code

Define a procedure to be accessed in the Maplet.

```
> GetColor:=proc()
local R, G, B, result;
use Maplets[Tools] in

# Convert the color value defined in the Color dialog
result:=Get(ColorDialog1);
# The result format is "#RRGGBB" in hexadecimal(base 16)
# Convert to values in the range 0..1
R:=convert(result[2..3], 'decimal', 16)/255;
G:=convert(result[4..5], 'decimal', 16)/255;
B:=convert(result[6..7], 'decimal', 16)/255;

# Plot the function entered in the MathMLEditor region
plot(MathML:-Import(Get(MathMLEditor1)), x=0..10,
color=COLOR(RGB, R, G, B));

end use;
end proc;
```

Load the **Elements** package.

```
> with(Maplets[Elements]):
```

Define the Maplet application.

```
> PlottingMaplet2:= Maplet(
    'onstartup' = Action(RunWindow(Window1)),
Window('reference' = Window1,
  BoxLayout(
    BoxColumn(
      BoxRow(
        Plotter('reference' = Plotter1)),
      BoxRow(
        MathMLEditor('reference' = MathMLEditor1)),
      BoxRow(
# Access the GetColor procedure and plot the result
        Button("Plot", Evaluate('function' = 'GetColor',
          'target' = 'Plotter1')),
# Launch the Color dialog
        Button("Color", RunDialog('dialog' = 'ColorDialog1')),
# Close the Maplet
        Button("Close", Shutdown()))
      )
    )
  ),
Action('reference' = 'approveColorDialog1'),
Action('reference' = 'cancelColorDialog1'),
ColorDialog('onapprove' = 'approveColorDialog1',
  'oncancel' = 'cancelColorDialog1',
  'reference' = 'ColorDialog1')
);
```

Display the Maplet

```
> Maplets[Display](PlottingMaplet2);
```

For more information on the **Maplets** package, refer to the **?MapletsPackage** help page. For more examples of designing Maplets using the **Maplets** package, see the **?Maplets/Roadmap** help page.

## Saving

When saving a Maplet, you can save the document as an **.mw** file or you can export the document as a **.maplet** file.

### Maple Document

To save the Maplet code as an **.mw** file:

1. From the **File** menu, select **Save**.
2. Navigate to the save location.
3. Enter a filename.
4. Click **Save**.

If the document contains only Maplet code, it is recommended that you export the document as a **.maplet** file.

### Maplet File

To export the Maplet code as a **.maplet** file:

1. From the **File** menu, select **Export As**.
2. In the **Files of Type** drop-down list, select **Maplet**.
3. Navigate to the export location.
4. Enter the filename.
5. Click **Save**.



# 10 Input, Output, and Interacting with Other Products

## 10.1 In This Chapter

| Section                                                                                             | Topics                                                                                                                                                                                                    |
|-----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Writing to Files</b> - Saving to Maple file formats                                              | <ul style="list-style-type: none"><li>• Saving Data to a File</li><li>• Saving Expressions to a File</li></ul>                                                                                            |
| <b>Reading from Files</b> - Opening Maple files                                                     | <ul style="list-style-type: none"><li>• Reading Data from a File</li><li>• Reading Expressions from a File</li></ul>                                                                                      |
| <b>Exporting to Other Formats</b> - Exporting documents in file formats supported by other software | <ul style="list-style-type: none"><li>• Exporting Documents</li><li>• MapleNet</li><li>• Maple T.A.</li></ul>                                                                                             |
| <b>Connectivity</b> - Using Maple with other programming languages and software                     | <ul style="list-style-type: none"><li>• Translating Maple Code to Other Programming Languages</li><li>• Accessing External Products from Maple</li><li>• Accessing Maple from External Products</li></ul> |

## 10.2 Writing to Files

Maple supports file formats in addition to the standard **.mw** file format.

After using Maple to perform a computation, you can save the results to a file for later processing with Maple or another program.

### Saving Data to a File

If the result of a Maple calculation is a long list or a large array of numbers, you can convert it to Matrix form and write the numbers to a file using the **ExportMatrix** command. This command writes columns of numerical data



to a file, allowing you to import the numbers into another program. To convert a list or a list of lists to a **Matrix**, use the **Matrix** constructor. For more information, refer to the **?Matrix** help page.

```
> L := 
$$\begin{bmatrix} -81 & -98 & -76 & -4 & 29 \\ -38 & -77 & -72 & 27 & 44 \\ -18 & 57 & -2 & 8 & 92 \\ 87 & 27 & -32 & 69 & -31 \\ 33 & -93 & -74 & 99 & 67 \end{bmatrix} :$$

```

```
> ExportMatrix("matrixdata.txt", L):
```

If the data is a Vector or any object that can be converted to type Vector, use the **ExportVector** command. To convert lists to Vectors, use the **Vector** constructor. For more information, refer to the **?Vector** help page.

```
> R := [3, 3.1415, -65, 0]
```

```
R := [3, 3.1415, -65, 0]
```

```
> V := Vector(R)
```

```
V := 
$$\begin{bmatrix} 3 \\ 3.1415 \\ -65 \\ 0 \end{bmatrix}$$

```

```
> ExportVector("vectordata.txt", V):
```

You can extend these routines to write more complicated data, such as complex numbers or symbolic expressions. For more information, refer to the **?ExportMatrix** and **?ExportVector** help pages.

For more information on matrices and vectors, see *Linear Algebra (page 135)*.

## Saving Expressions to a File

If you construct a complicated expression or procedure, you can save them for future use in Maple. If you save the expression or procedure in the Maple internal format, Maple can retrieve it more efficiently than from a document. Use the **save** command to write the expression to a **.m** file. For more information on Maple internal file formats, refer to the **?file** help page.

$$> \text{qbinomial} := (n, k) \rightarrow \frac{\prod_{i=n-k+1}^n (1-q^i)}{\prod_{i=1}^k (1-q^i)};$$

In this example, small expressions are used. In practice, Maple supports expressions with thousands of terms.

> `expr := qbinomial(10, 4)`

$$\text{expr} := \frac{(1-q^7)(1-q^8)(1-q^9)(1-q^{10})}{(1-q)(1-q^2)(1-q^3)(1-q^4)}$$

> `nexpr := normal(expr)`

$$\text{nexpr} := \frac{(q^6 + q^5 + q^4 + q^3 + q^2 + q + 1)(q^4 + 1)}{(q^6 + q^3 + 1)(q^8 + q^6 + q^4 + q^2 + 1)}$$

You can save these expressions to the file **qbinom.m**.

> `save qbinomial, expr, nexpr, "qbinom.m"`

Clear the memory using the **restart** command and retrieve the expressions using the **read** command.

> *restart*

> *read*"qbinom.m"

> *expr*

$$\frac{(1-q^7)(1-q^8)(1-q^9)(1-q^{10})}{(1-q)(1-q^2)(1-q^3)(1-q^4)}$$

For more information on writing to files, refer to the **?save** help page.

## 10.3 Reading from Files

The most common reason for reading files is to load data, for example, data generated in an experiment. You can store data in a text file, and then read it into Maple using the **Import Data Assistant**.

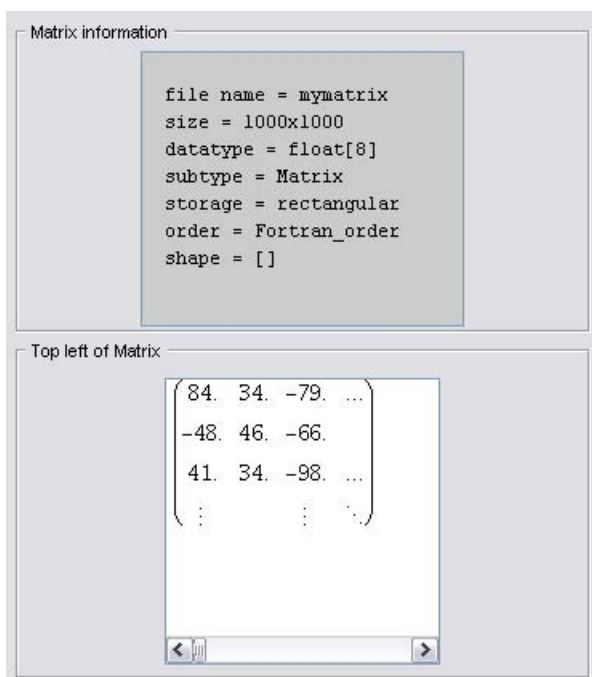
### Reading Data from a File

#### Import Data Assistant

If you generate data outside Maple, you must read it into Maple before manipulating it. This data can be an image, a sound file, or columns of numbers in a text file. You can import this external data into Maple using the **Import Data Assistant**.

#### To launch the Import Data Assistant:

- From the **Tools** menu, select **Assistants**, and then **Import Data**.
- Using the **Select Data Source** dialog, select the data file to import.



**Figure 10.1: Import Data Assistant (Detail)**

From the main window, you can preview the selected file, and specify the source format, source form, and behavior on close. You can also select a different file to be imported. Additional help is available from the **Help** menu of the **Import Data** window.

### ImportMatrix Command

The **Import Data Assistant** provides a graphical interface to the **ImportMatrix** command. For more information, including options not available in the assistant, refer to the **?ImportMatrix** help page.

## Reading Expressions from a File

You can write Maple programs in a text file using a text editor, and then import the file into Maple. You can paste the commands from the text file into your document or you can use the **read** command.

When you read a file with the **read** command, Maple treats each line in the file as a command. Maple executes the commands and displays the results in your document but it does *not*, by default, insert the commands from the file in your document.

For example, the file **ks.tst** contains the following Maple commands.

```
S:= n -> sum( binomial( n, beta ) * ( ( 2*beta )! / 2^beta - beta!*beta ),  
beta=1..n );
```

```
S(19);
```

When you read the file, Maple displays the results but not the commands.

```
> read "ks.tst"
```

$$S := n \rightarrow \sum_{\beta=1}^n \text{binomial}(n, \beta) \left( \frac{(2\beta)!}{2^\beta} - \beta! \beta \right)$$

1024937361666644598071114328769317982974

If you set the **interface echo** option to 2, Maple inserts the commands from the file into your document.

```
> interface(echo = 2);
```

```
> read "ks.tst"
```

```
>
```

```
S:=n->sum(binomial(n,beta)*((2*beta)!/2^beta-beta!*beta),beta=1..n);
```

$$S := n \rightarrow \sum_{\beta=1}^n \text{binomial}(n, \beta) \left( \frac{(2\beta)!}{2^\beta} - \beta! \beta \right)$$

```
> S(19);
```

1024937361666644598071114328769317982974

For more information, refer to the **?read** and **?interface** help pages.

## 10.4 Exporting to Other Formats

### Exporting Documents

You can save your documents by selecting **Save** or **Save As** from the **File** menu. By selecting **Export As** from the **File** menu, you can also export a document in the following formats: HTML, LaTeX, Maple input, Maplet application, Maple text, plain text, and Rich Text Format. This allows you to access your work outside Maple.

#### HTML

The **.html** file that Maple generates can be loaded into any HTML browser. Exported mathematical content can be displayed in one of the following formats: GIF, MathML 2.0 Presentation, MathML 2.0 Content, or Maple Viewer, and is saved in a separate folder. MathML is the Internet standard, sanctioned by the World Wide Web Consortium (W3C), for the communication of structured mathematical formulae between applications. For more information about MathML, refer to the **?MathML** help page.

Maple documents that are exported to HTML translate into multiple documents when using frames. If the frames feature is not selected, Maple creates only one page that contains the document contents.

#### LaTeX

The **.tex** file generated by Maple is ready for processing by LaTeX. All distributions of Maple include the necessary style files. By default, the LaTeX style files are set for printing the **.tex** file using the **dvips** printer driver. You can change this behavior by specifying an option to the **\usepackage** LaTeX command in the preamble of your **.tex** file. For more information, refer to the **?exporttoLaTeX** help page.

## Maple Input

You can export a Maple document as Maple input so that it can be loaded using the Maple Command-line version.

**Important:** When exporting a document as Maple input for use in Command-line Maple, your document must contain explicit semicolons in 1-D Math input. If not, the exported **.mpl** file will not contain semicolons, and Command-line Maple will generate errors.

## Maplet Application

The **Export as Maplet** facility saves a Maple document as a **.maplet** file, so that you can run it using the command-line interface or the **MapletViewer**. The MapletViewer is an executable program that can launch saved Maplet applications. It displays and runs Maplet applications independently of the Maple Worksheet interface.

**Important:** When exporting a document as a Maplet Application for use in Command-line Maple or the MapletViewer, your document must contain explicit semicolons. If not, the exported **.maplet** file will not contain semicolons, and Command-line Maple and the MapletViewer will generate errors.

For information on using the MapletViewer, see *Using Maplets* (page 344).

## Maple Text

Maple text is marked text that retains the distinction between text, Maple input, and Maple output. Thus, you can export a document as Maple text, send the text file by email, and the recipient can import the Maple text into a Maple session and regenerate the computations in the original document.

## Plain Text

Export a Maple document as plain text so that you can open the text file in another application.

## Rich Text Format (RTF)

The **.rtf** file generated by Maple can be loaded into any word processor that supports **RTF**.

## Summary of Translation

**Table 10.1: Summary of Content Translation When Exporting to Different Formats**

| Content                      | HTML          | LaTeX                | Maple Input            | Maplet Application     | Maple Text                              | Plain Text                              | Rich Text Format      |
|------------------------------|---------------|----------------------|------------------------|------------------------|-----------------------------------------|-----------------------------------------|-----------------------|
| Text                         | Maintained    | Maintained           | Preceded by #          | Preceded by #          | Preceded by #                           | Maintained                              | Maintained            |
| 1-D Math                     | Maintained    | Maintained           | Maintained             | Maintained             | Preceded by >                           | Preceded by >                           | Static image          |
| 2-D Math                     | GIF or MathML | 1-D Math or LaTeX 2e | 1-D Math (if possible) | 1-D Math (if possible) | 1-D Math or character-based typesetting | 1-D Math or character-based typesetting | Static image          |
| Plot                         | GIF           | Postscript file      | Not exported           | Not exported           | Not exported                            | Not exported                            | Static image          |
| Animation                    | Animated GIF  | Not exported         | Not exported           | Not exported           | Not exported                            | Not exported                            | Not exported          |
| Hidden content               | Not exported  | Not exported         | Not exported           | Not exported           | Not exported                            | Not exported                            | Not exported          |
| Manually inserted page break | Not supported | Not supported        | Not supported          | Not supported          | Not supported                           | Not supported                           | RTF page break object |



| Content                         | HTML                                                                                              | LaTeX                                                 | Maple Input  | Maplet Application | Maple Text   | Plain Text   | Rich Text Format |
|---------------------------------|---------------------------------------------------------------------------------------------------|-------------------------------------------------------|--------------|--------------------|--------------|--------------|------------------|
| Hyperlink                       | Links to help pages become plain text. Links to documents are renamed and converted to HTML links | Plain text                                            | Plain text   | Plain text         | Plain text   | Plain text   | Plain text       |
| Embedded image or sketch output | GIF                                                                                               | Not exported                                          | Not exported | Not exported       | Not exported | Not exported | Static image     |
| Spreadsheet                     | HTML table                                                                                        | LaTeX tables                                          | Not exported | Not exported       | Not exported | Not exported | RTF table        |
| Document style                  | Approximated by HTML style attributes                                                             | LaTeX environments and sections, LaTeX 2e macro calls | Not exported | Not exported       | Not exported | Not exported | RTF style        |

## MapleNet

### Overview of MapleNet

Using MapleNet, you can deploy Maple content on the Web. Powered by the Maple computation engine, MapleNet allows you to embed dynamic formulas, models, and diagrams as **live** content in Web pages. The MapleNet software is not included with the Maple software. For more information on MapleNet, visit <http://www.maplesoft.com/maplenet>.

## MapleNet Documents and Maplets

After you upload your Maple documents to the MapleNet server, it can be accessed by anyone in the world using a Web browser. Even if viewers do not have a copy of Maple installed, they can view documents and Maplets, manipulate 3-D plots, and execute code at the click of a button.

## Custom Java™ Applets and JavaServer Pages™ Technology

MapleNet provides a programming interface to the Maple math engine so commands can be executed from a Java™ applet or using JavaServer Pages™ technology. Embed MapleNet into your Web application, and let Maple handle the math and visualization.

## Maple T.A.

### Overview of Maple T.A.

Maple T.A. is a Web-based automated testing system, based on the Maple engine. Instructors can use pre-written questions or create custom question banks and then choose from these questions to form quizzes and assignments. Maple T.A. automatically grades responses as students complete assignments and tests. For more information, visit <http://www.maplesoft.com/mapleta>.

### Exporting Assignments to Maple T.A.

You can use Maple to create graded questions for use in Maple T.A. For information on creating and testing questions, see *Creating Graded Assignments* (page 271). Using the Maple T.A. export feature, you can create and test Maple T.A. content.

#### To export the document:

1. From the **File** menu, select **Export As**.
2. In the **Export As** dialog, specify a filename and the **Maple T.A. (.zip)** file type. The .zip file containing your questions and assignment can be uploaded to Maple T.A. as a course module.

Any document content outside Maple T.A. sections (indicated by green section markers) is ignored by the export process.

For more details, refer to the `?exporttoMapleTA` help page.

## 10.5 Connectivity

### Translating Maple Code To Other Programming Languages

#### Code Generation

The **CodeGeneration** package is a collection of commands and subpackages that enable the translation of Maple code to other programming languages. Languages currently supported include C, Fortran77, Java, MATLAB®, and Visual Basic®.

For details on Code Generation, refer to the `?CodeGeneration` help page.

### Accessing External Products from Maple

#### External Calling

External calling allows you to use compiled C, Fortran77, or Java code in Maple. Functions written in these languages can be linked and used as if they were Maple procedures. With external calling you can use pre-written optimized algorithms without the need to translate them into Maple commands. Access to the NAG library routines and other numerical algorithms is built into Maple using the external calling mechanism.

External calling can also be applied to functions other than numerical algorithms. Routines exist that accomplish a variety of non-mathematical tasks. You can use these routines in Maple to extend its functionality. For example, you can link to controlled hardware via a serial port or interface with another program. The **Database Integration Toolbox** uses external calling to allow you to query, create, and update databases in Maple. For more information, visit <http://www.maplesoft.com/products/toolboxes>.

For more information on using external calling, refer to the **?ExternalCalling** help page.

## **Mathematica Translator**

The **MmaTranslator** package provides translation tools for converting Mathematica® expressions, command operations, and notebooks to Maple. The package can translate Mathematica input to Maple input and Mathematica notebooks to Maple documents. The **Mma** subpackage contains commands that provide translation for Mathematica commands when no equivalent Maple command exists. In most cases, the command achieves the translation through minor manipulations of the input and output of similar Maple commands.

**Note:** The **MmaTranslator** package does not convert Mathematica programs.

There is a Maplet interface to the **MmaTranslator** package. For more information, refer to the **?MmaToMaple** help page.

## **Matlab Package**

The **Matlab** package enables you to call selected MATLAB functions from a Maple session, provided you have MATLAB installed on your system.

For more information, refer to the **?Matlab** help page.

## **Accessing Maple from External Products**

### **Microsoft Excel Add-In**

Maple is available as an add-in to Microsoft Excel 2000, Excel 2003, and Excel XP for Windows, and provides the following features.

- Access to Maple commands from Excel
- Ability to copy and paste between Maple and Excel
- Access to a subset of the Maple help pages


- Maple Function Wizard to step you through the creation of a Maple function call

**To enable the Maple Excel Add-in in Excel 2000, Excel 2003, or Excel XP:**

1. From the **Tools** menu, choose **Add-Ins**.
2. If the Maple Excel Add-in is not listed:
  - Click **Browse** and navigate to the directory in which Maple is installed.
  - In the **Excel** directory, select the **WMIMPLEX.xla** file.
  - Click **OK**.
3. Select the **Maple Excel Add-in** check box.
4. Click **OK**.

More information is available in the **Using Maple in Excel** online help file within Excel.

**To view this help file:**

1. Enable the add-in.
2. From the **View** menu, select **Toolbars**, and then **Maple**.
3. On the Maple toolbar, click the Maple help icon .

**OpenMaple**

OpenMaple is a suite of functions that allows you to access Maple algorithms and data structures in your compiled C, Java, or Visual Basic programs. (This is the reverse of external calling, which allows access to compiled C, Fortran77, and Java code from Maple.)

To run your application, Maple 9 or later must be installed. You can distribute your application to any licensed Maple 9 or later user. For additional terms and conditions on the use of OpenMaple, refer to **extern/OpenMapleLicensing.txt** in your Maple installation.

For more details on using OpenMaple functions, refer to the **?OpenMaple** help page.



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